Matematický pohled na hudební teorii

Ada Böhmová

ada@kreatrix.org
github.com/spirali

1. Nic mi nevěřte

1. Nic mi nevěřte
2. Hudební teorii nepotřebujete

1. Nic mi nevěřte
2. Hudební teorii nepotřebujete
3. Pravděpodobně máte hudební sluch

European Journal of Human Genetics (2017) 25, 625–630
© 2017 Macmillan Publishers Limited, part of Springer Nature. All rights reserved 1018-4813/17
www.nature.com/ejhg

ARTICLE

Prevalence of congenital amusia

Isabelle Peretz* and Dominique T Vuvan

Congenital amusia (commonly known as tone deafness) is a lifelong musical disorder that affects 4% of the population according to a single estimate based on a single test from 1980. Here we present the first large-based measure of prevalence with a sample of 20 000 participants, which does not rely on self-referral. On the basis of three objective tests and a questionnaire, we show that (a) the prevalence of congenital amusia is only 1.5%, with slightly more females than males, unlike other developmental disorders where males often predominate; (b) self-disclosure is a reliable index of congenital amusia, which suggests that congenital amusia is hereditary, with 46% first-degree relatives similarly affected; (c) the deficit is not attenuated by musical training and (d) it emerges in relative isolation from other cognitive disorder, except for spatial orientation problems. Hence, we suggest that congenital amusia is likely to result from genetic variations that affect musical abilities specifically.
European Journal of Human Genetics (2017) **25,** 625–630; doi:10.1038/ejhg.2017.15; published online 22 February 2017


MÁTE MATEMATIKU?
A MOHLA BYCH JI VIDĚT?
imgflip.com


Steinway & Sons
Patent Grand
New York & Hamburg
C-227

Why does the scale have seven (or five) notes? Why not six?

Asked 10 years, 3 months ago Modified 6 years, 5 months ago Viewed 30k times

50

I'm a programmer by trade, and I have always felt that music was arbitrarily difficult. Please forgive my inexperience with musical notation. I had a little thought experiment with my wife today, and I wanted to ask why we don't do it the way I thought up.

My wife explained to me that a scale(octave?) is made up of seven notes, which we typically call ABCDEFG or Do-Re-Mi-Fa-So-La-Ti(-Do). From this answer: https://music.stackexchange.com/a/3004 we know that those 7(8) notes are this progression:

> Every major scale has seven notes. They all start on a root note and proceed to go up in the following pattern: Whole Step, Whole Step, *Half Step*, Whole Step, Whole Step, Whole Step, and then a final *Half Step* returns to the root note (an octave above where we started).

Why go up by a half step twice? Why not go up a whole step every time? It seems like having B# be C and Cb be B (and same with E/F) is arbitrarily complicated. Was this done just to make pianos easier to play by feel? Is there a mathematical root?

If you will suspend your disbelief with me for a minute, what if we had a scale made up of 7 lines? The spaces in between each line represent the notes (I'll call them 1-6, to avoid confusion with A-G). The lines themselves represent sharps and flats. So a 1# is a 2b, etc.

The piano would have to change to having black keys in between every white key. To offset this, the 1 keys would be wider on the left, and the 6 keys would be wider on the right so that one could still determine octaves (septaves?) by feel.

What problems does this present? Is there a good reason not to go to an easier to remember system? If not, why has no one done it?

25

I think your question is largely about the chosen notation for the Western system, which most answers haven't really addressed.

The notation we have is actually pretty natural and logical, for a simple reason: there are twelve different notes in the Western system, but only a subset of these -- seven, in fact -- are used in a given scale such as the major scale.

Let's use individual semitones as the basis for a notation as you suggest; so, let's say the note A is still denoted by A, but now A# (or Bb) is denoted by B, and then the remaining notes are C, D, E, F, G, H, I, J, K, and L (twelve in total).

I understand why you'd want to do this; it removes synonyms. But at what cost? What does an actual key look like now? Take C major as an example. In the new notation, the notes are D, F, H, I, K, A, C. This is confusing and hard to remember. Compare with C major in normal notation: C, D, E, F, G, A, B. It just cycles through the seven letters.

What about other keys? Let's take F major as another example. I won't write it all out in the new notation again because you just get another confusing list of letters, but in normal notation, it's F, G, A, Bb, C, D, E.

Hopefully now you see the benefit of this notation: it's easy to think about every key, because, ignoring accidentals (i.e. the flat on the B) they just cycle through our seven letters.

You lose uniqueness of note names -- though in fact, not really in practice, for example you'd never call Bb "A#" when talking about the F major key -- and the usefulness of this feature of the notation far outweighs this minor problem.

57

You can divide up the octave however you want, but it turns out that doing what you suggest doesn't really make good sounding music, at least to our western ears.

It all has to do with overtones and pleasant ratios of pitches. An interval sounds consonant to us when the ratio of the frequencies is mathematically simple. It causes the waveforms line up and produce constructive interference.

If I take C as a base from which to construct the overtone series, I quickly find G and E to have simple ratios (3:1 and 5:1, and by shifting octaves to get them closer together, 3:2 and 5:4). Stack two fifths and drop the octave to create D = 9:8, and go a fifth down and an octave up to create F = 4:3. Now we have the beginning of a scale: C D E F G, and the notes aren't evenly spaced (E-F is roughly half the distance of the others). This is the beginning of Pythagorean tuning, and various ways to construct the remaining notes of the major scale and fill in the gaps result in a huge number of ratio-based tunings.

In short: it's the way it is because it sounds good. Sure, it's a bit screwy in some ways, but we don't want to force an art form to conform to some notion of mathematical simplicity.

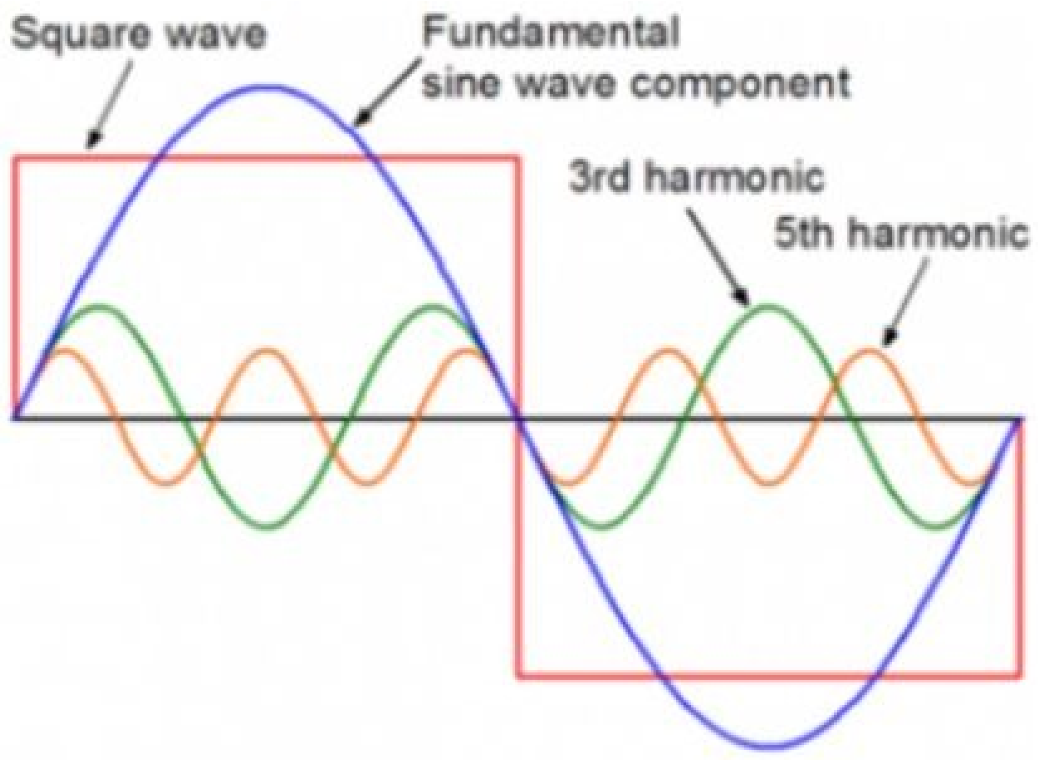
Share Improve this answer Follow

answered Jun 7, 2015 at 19:56
MattPutnam
22.9k 2 55 102

The reason is that dividing an octave into 12 notes sounds the best for a very mathematical reason! The frequency of each semi-tone is $2^{1/12}$ away from its neighbours.

Note	C × ?	Fraction	Note	C × ?	Fraction
C	1	1/1	C	2	2/1
C♯/D♭	1.059	18/17	B	1.888	17/9
D	1.122	9/8	A♯/B♭	1.782	16/9
D♯/E♭	1.189	6/5	A	1.682	5/3
E	1.260	5/4	G♯/A♭	1.587	8/5
F	1.335	4/3	G	1.498	3/2
F♯/G♭	1.414	7/5	F♯/G♭	1.414	10/7
G	1.498	3/2	F	1.335	4/3
G♯/A♭	1.587	8/5	E	1.260	5/4
A	1.682	5/3	D♯/E♭	1.189	6/5
A♯/B♭	1.782	16/9	D	1.122	9/8
B	1.888	17/9	C♯/D♭	1.059	18/17
C	2	2/1	C	1	1/1

Notice how each fraction on the right hand side (descending) is almost the inverse of the left hand side (ascending)? The difference is one of the numbers is doubled or halved each time. The smaller the two numbers are and the smaller the difference between them the better they sound to us. This is because the parts of the waveforms they produce agree very often.



Most of the answers here appear to be focusing on why we ended up with a seven note scale in western music.

This is a great area of inquiry; however, it is worth noting that whatever the answer to this question, **the seven note scale is a fundamentally arbitrary product of Western culture**.

Dissonance and harmony are culturally relative. The idea of the octave appears in *almost* every society; however, the way in which the octave is split and which combinations of frequencies are pleasing vary entirely by culture.

"Strictly speaking, there are no structural characteristics that have been identified in all known musical systems." - http://www.academia.edu/10684651/Cross-Cultural_Perspectives_on_Music_and_Musicality

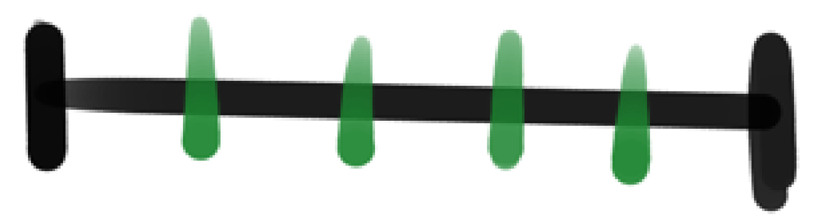
So I would argue that although the other answers are mostly correct in identifying reasons why we use a seven note scale, it should be kept in mind that these are fundamentally cultural and historical reasons, not biological or mathematical reasons.

Edit: Just wanted to disambiguate based on the comments. I am referring to the dictionary definition of "harmony," which is "the combination of different musical notes played or sung at the same time to produce a pleasing sound" - http://merriam-webster.com/dictionary/harmony. This definition is not related to any particular mathematical relationship or consonance between the notes: "Harmony" simply means that the resulting sound is pleasing to listener.

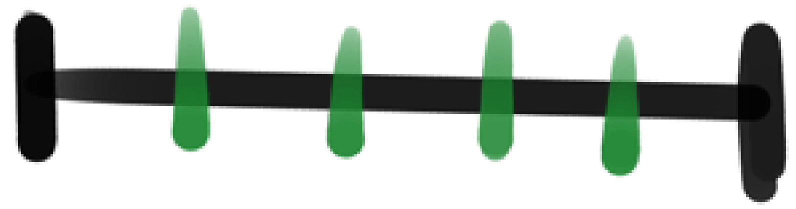
Spoiler alerts!

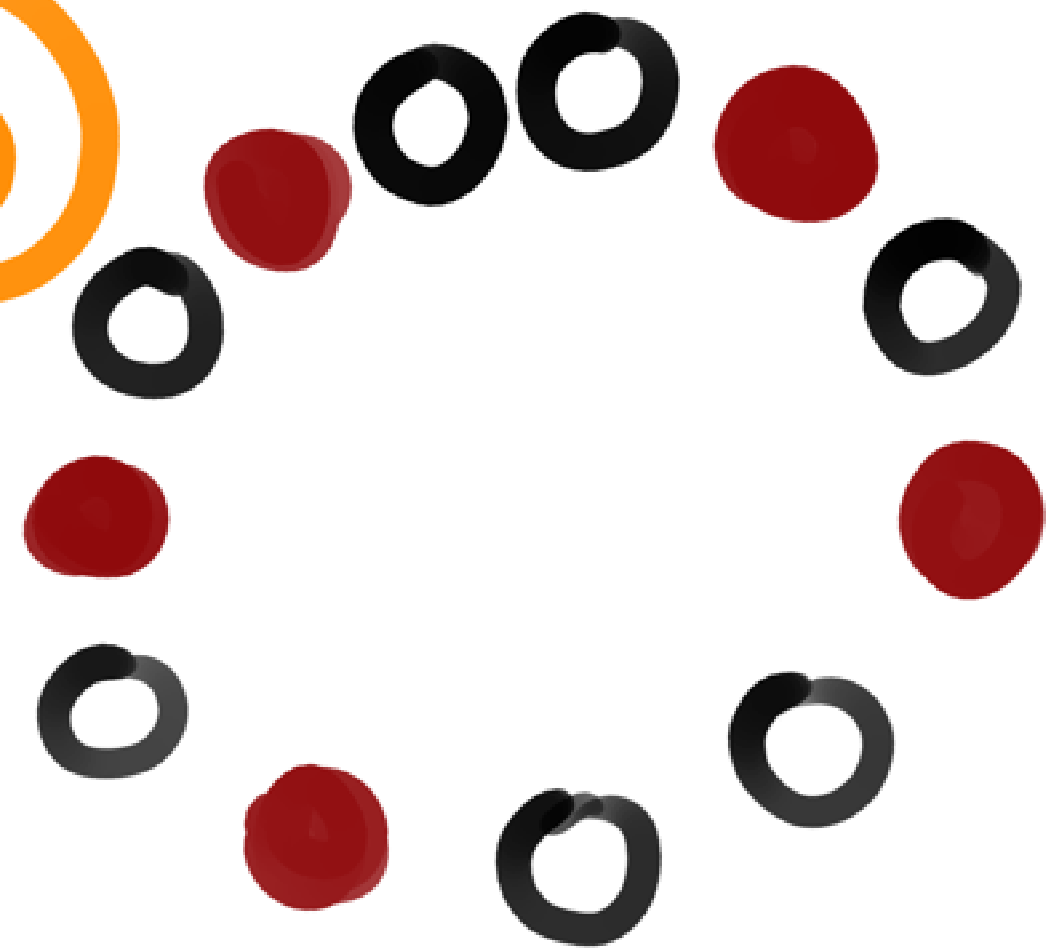
① $\frac{3}{2}$ $\frac{5}{3}$ $\frac{9}{8}$

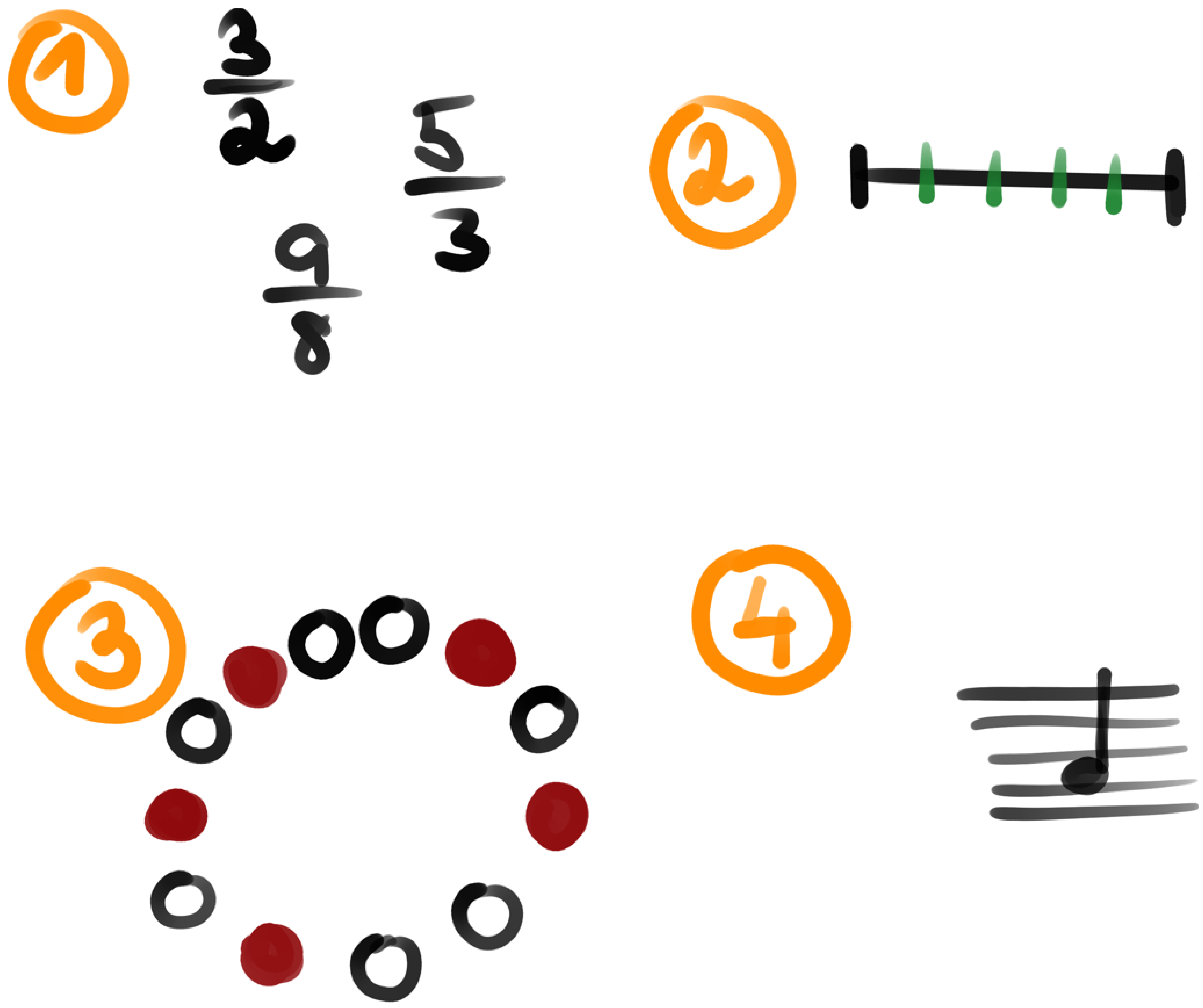
(1) $\frac{3}{2}$ $\frac{5}{3}$ $\frac{9}{8}$

(2) 

1. $\frac{3}{2}$ $\frac{5}{3}$ $\frac{9}{8}$

2. 

3. 


1
3/2
5/3
9/8
2
3
4

Sound is a vibration that travels as a wave through a medium (like air, water, or solids). It starts with something vibrating (e.g., vocal cords, guitar strings, speaker diaphragm). This creates alternating regions of compression and rarefaction, forming pressure waves that travel through the medium. Eventually, these waves reach a receiver (like your eardrum), which converts the vibrations into neural signals that your brain interprets as sound.

DEMO

wave

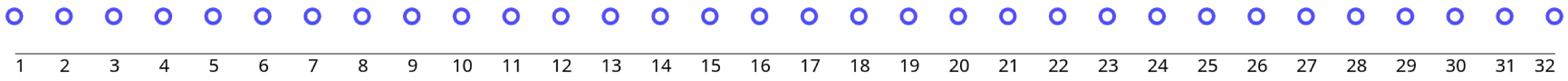
Principle 1: We perceive fundamental frequency as pitch

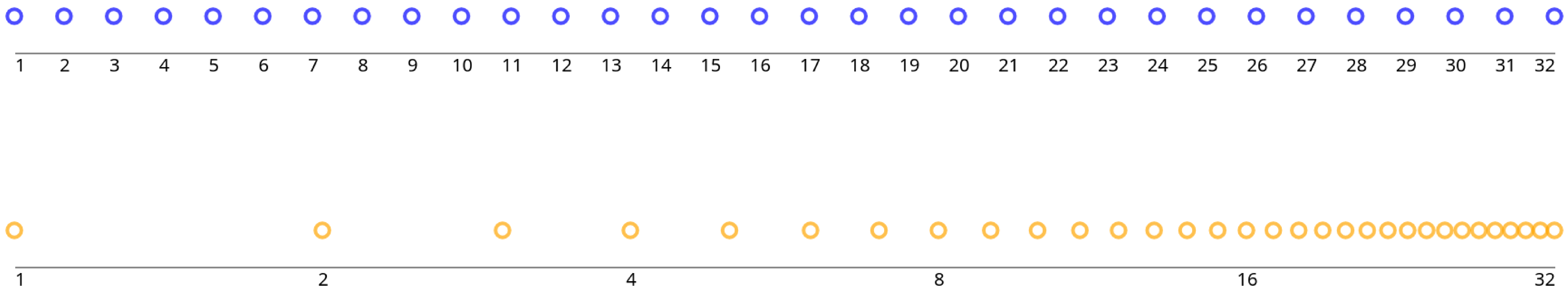
$$f, 2f, 3f, 4f, \ldots$$

DEMO

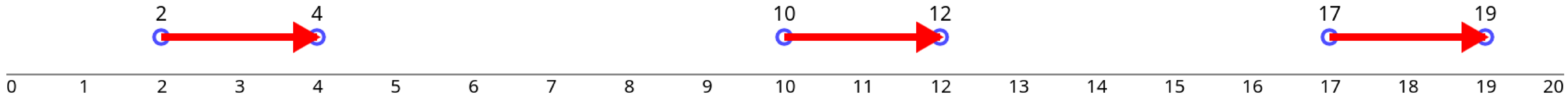
spectrum

Principle 2: Human hearing works on a logarithmic scale

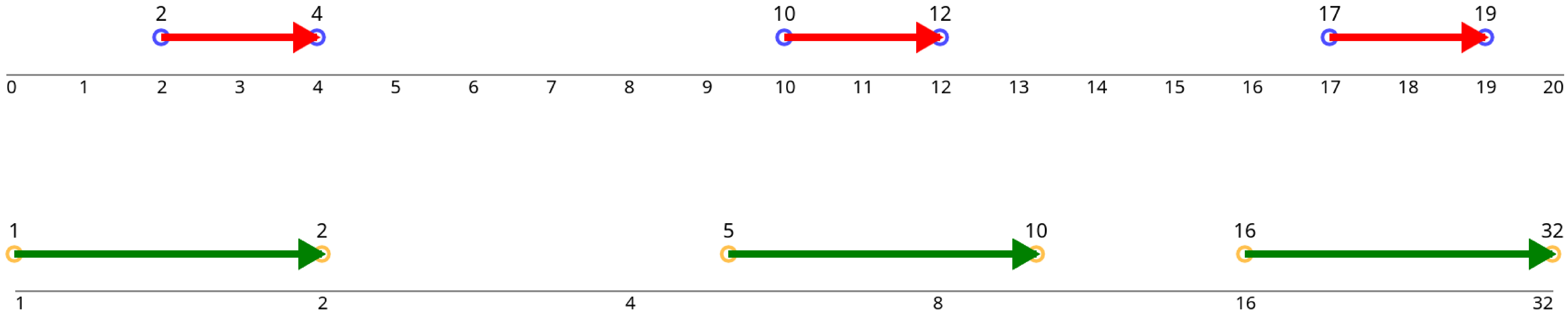

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32



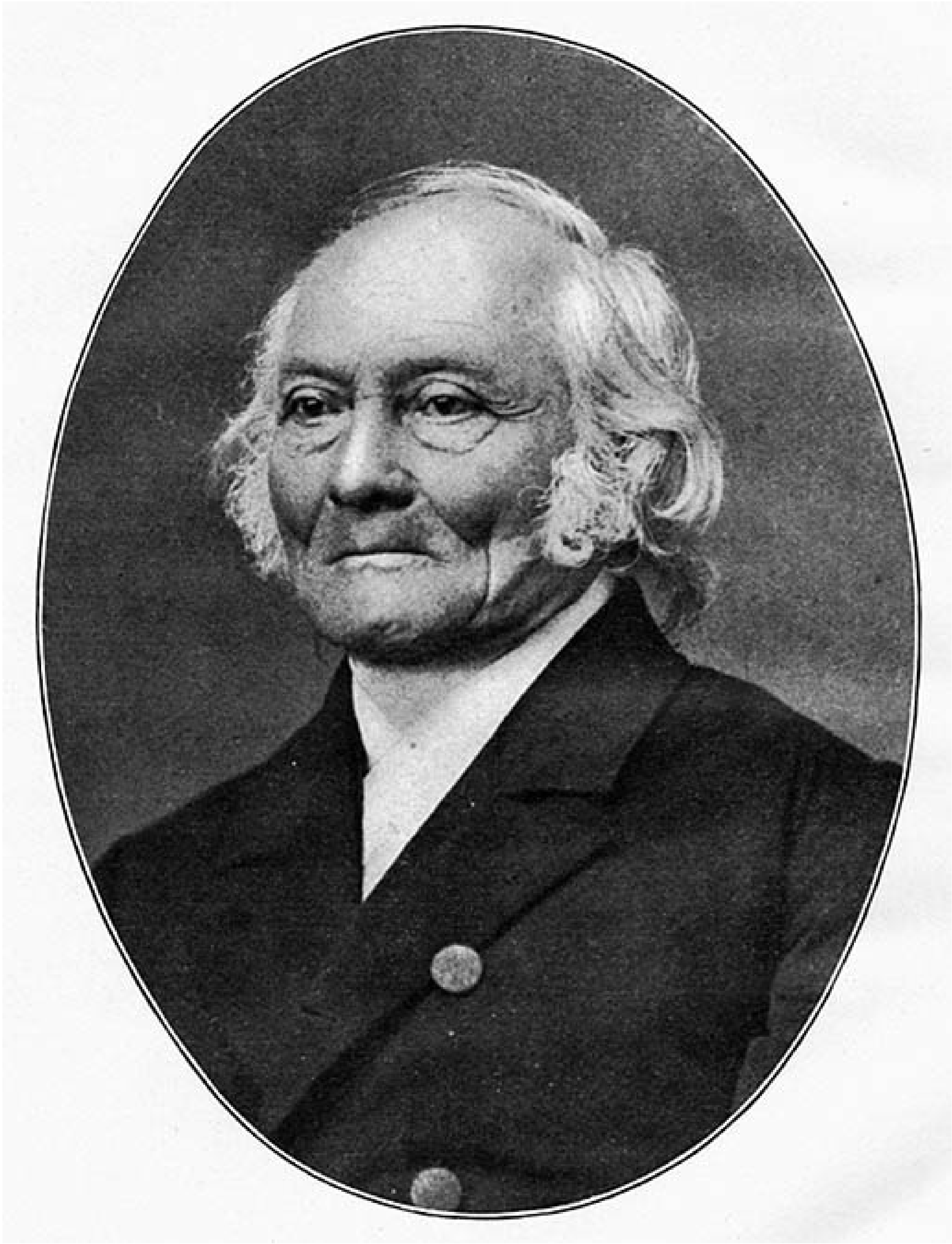
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32
1 2 4 8 16 32



2
4
10
12
17
19
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20


2
4
10
12
17
19
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
1
2
5
10
16
32
1 2 4 8 16 32

Weber–Fechner laws



Principle 3: Transpositional invariance

DEMO

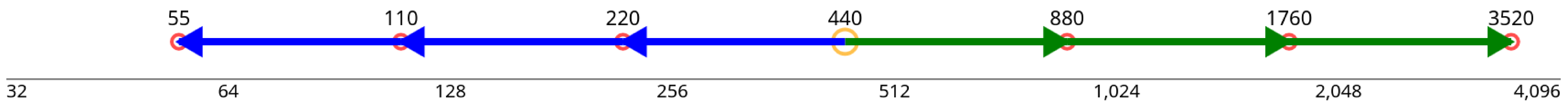
transpose

400
480
600
1000
1200
1500
400
600
800
1,000
1,200
1,400
1,600
1,800
2,000

Principle 4: Octave Circularity

1：2

1 : 2

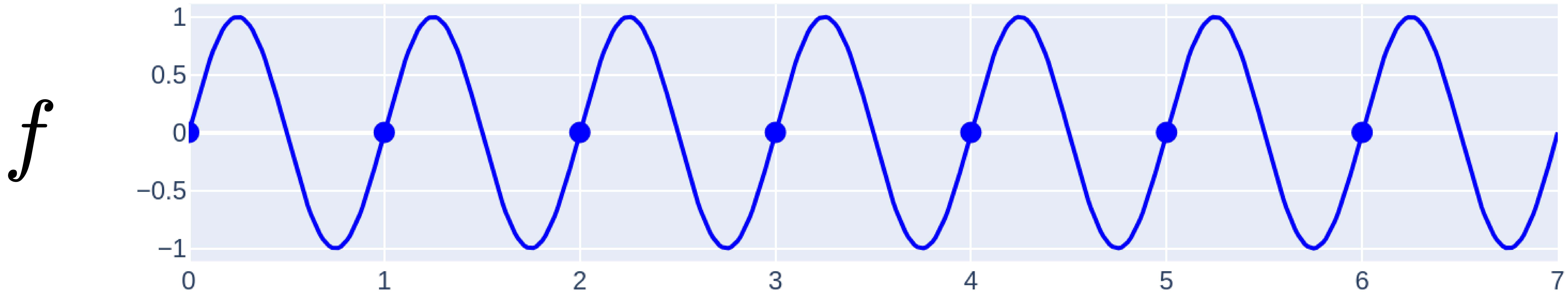


DEMO

octave

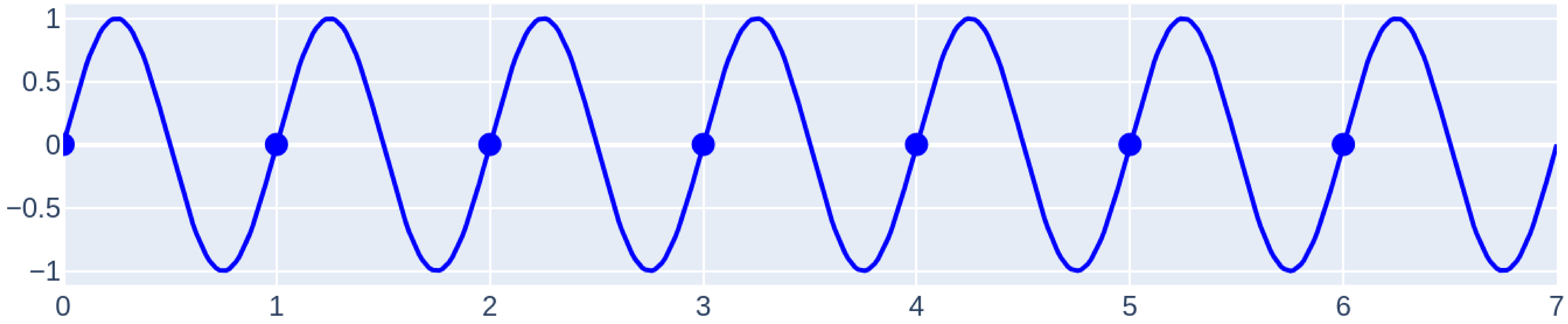
Principle 5: "Small ratios" sound good together

$$\frac{a}{b}$$

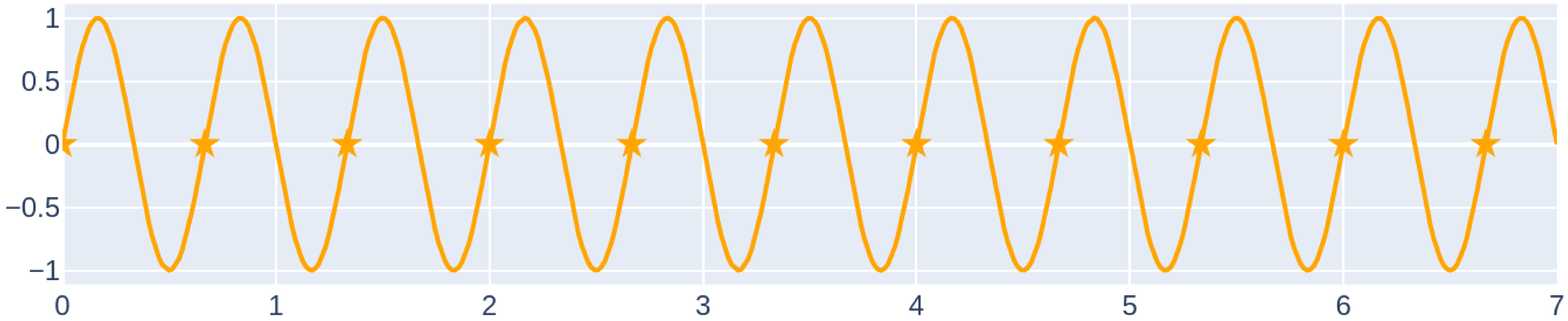


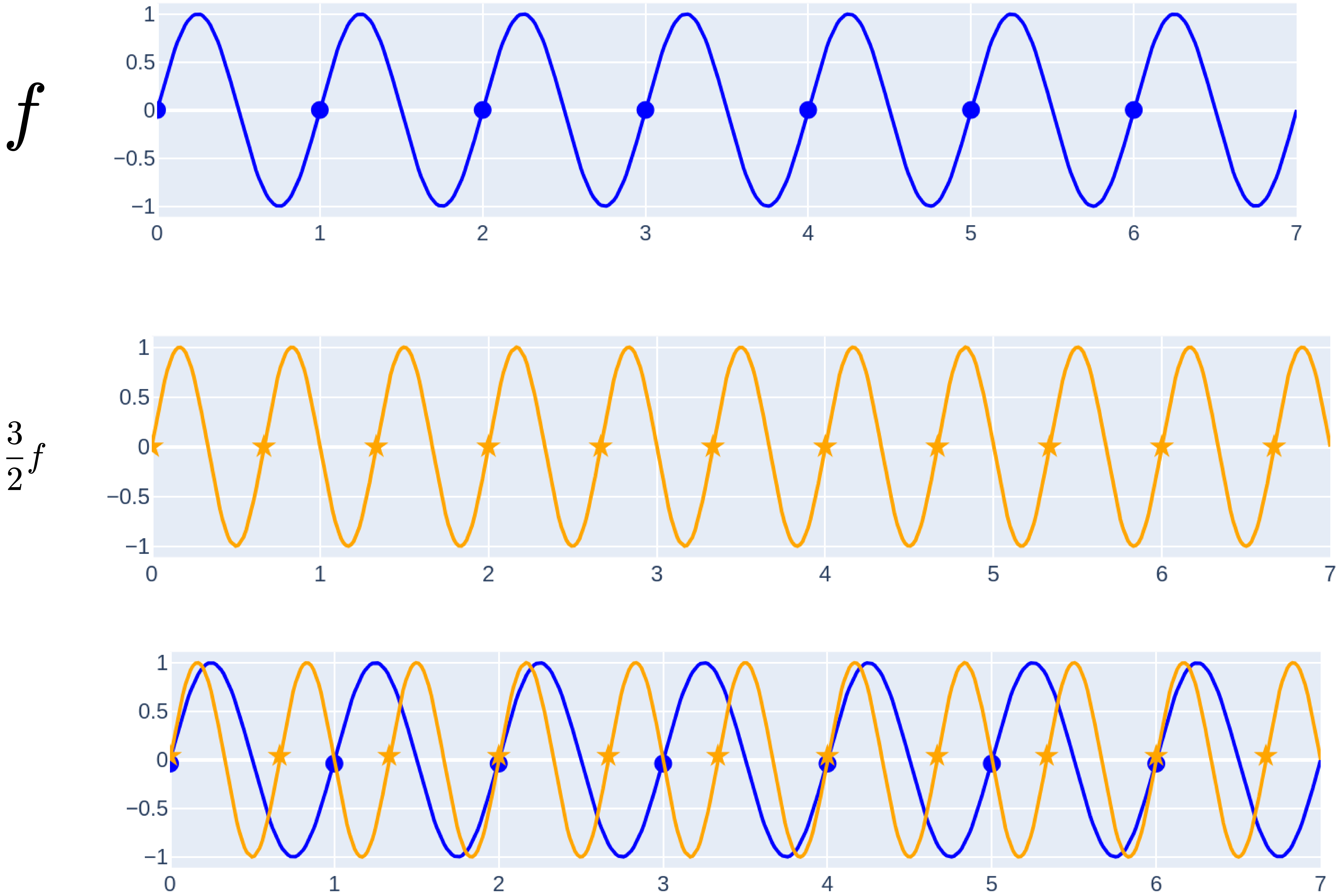
f
1
0.5
0
−0.5
−1
0
1
2
3
4
5
6
7

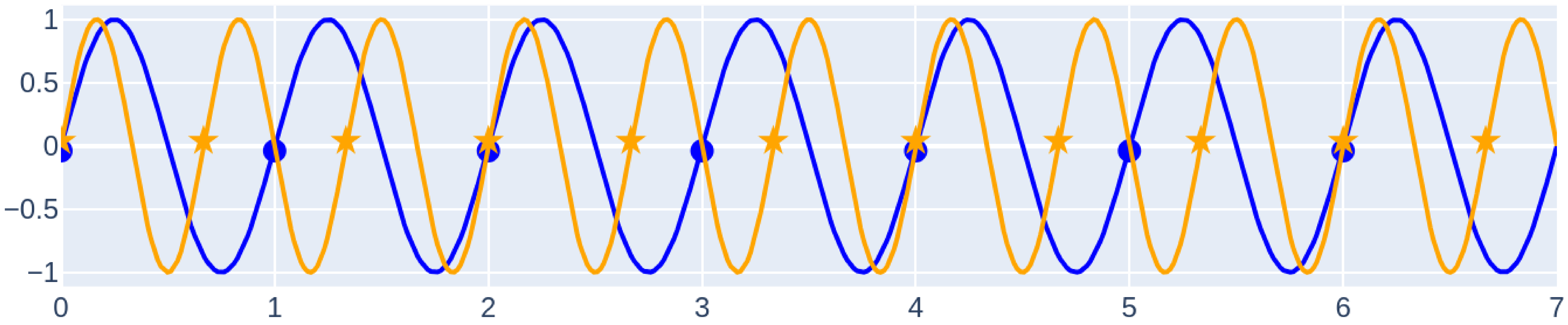
f

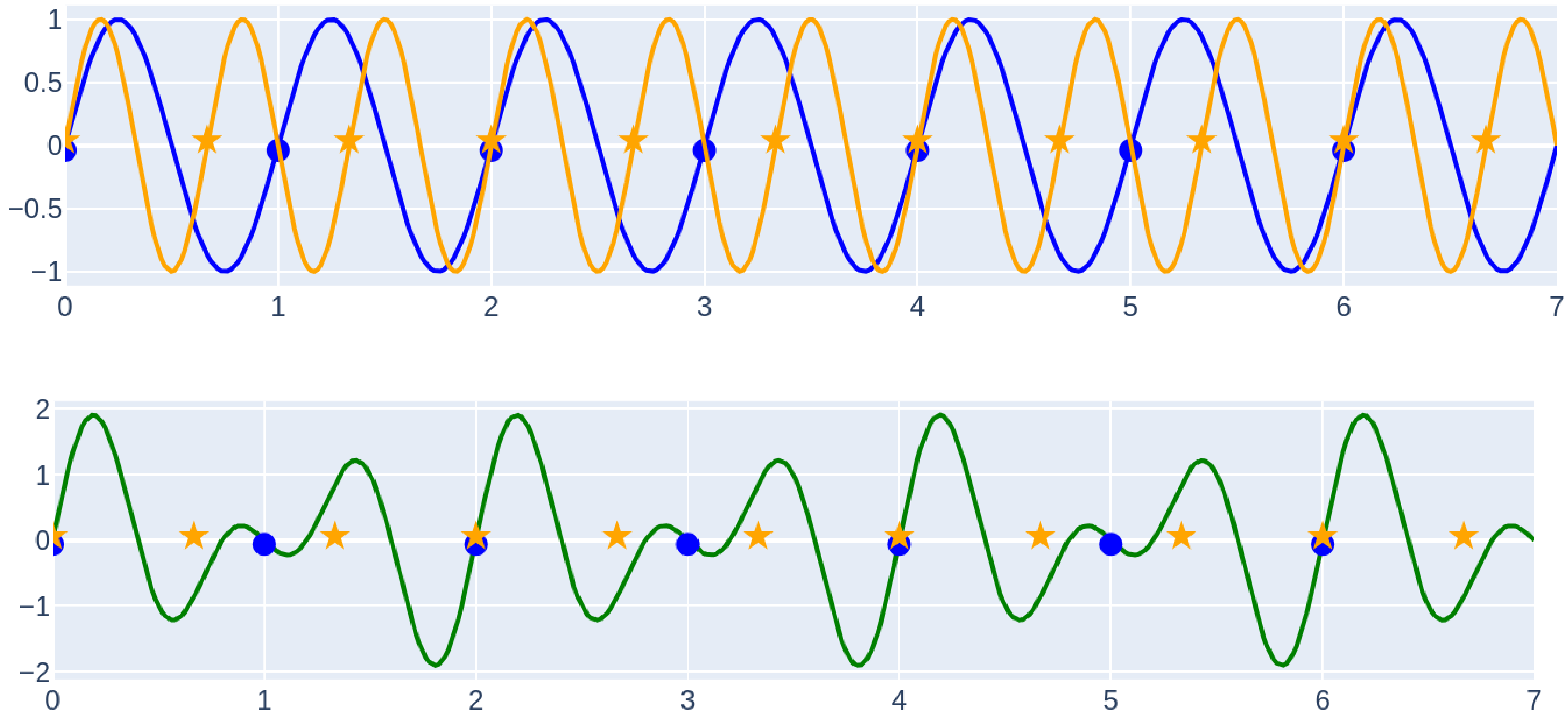


$\frac{3}{2}f$

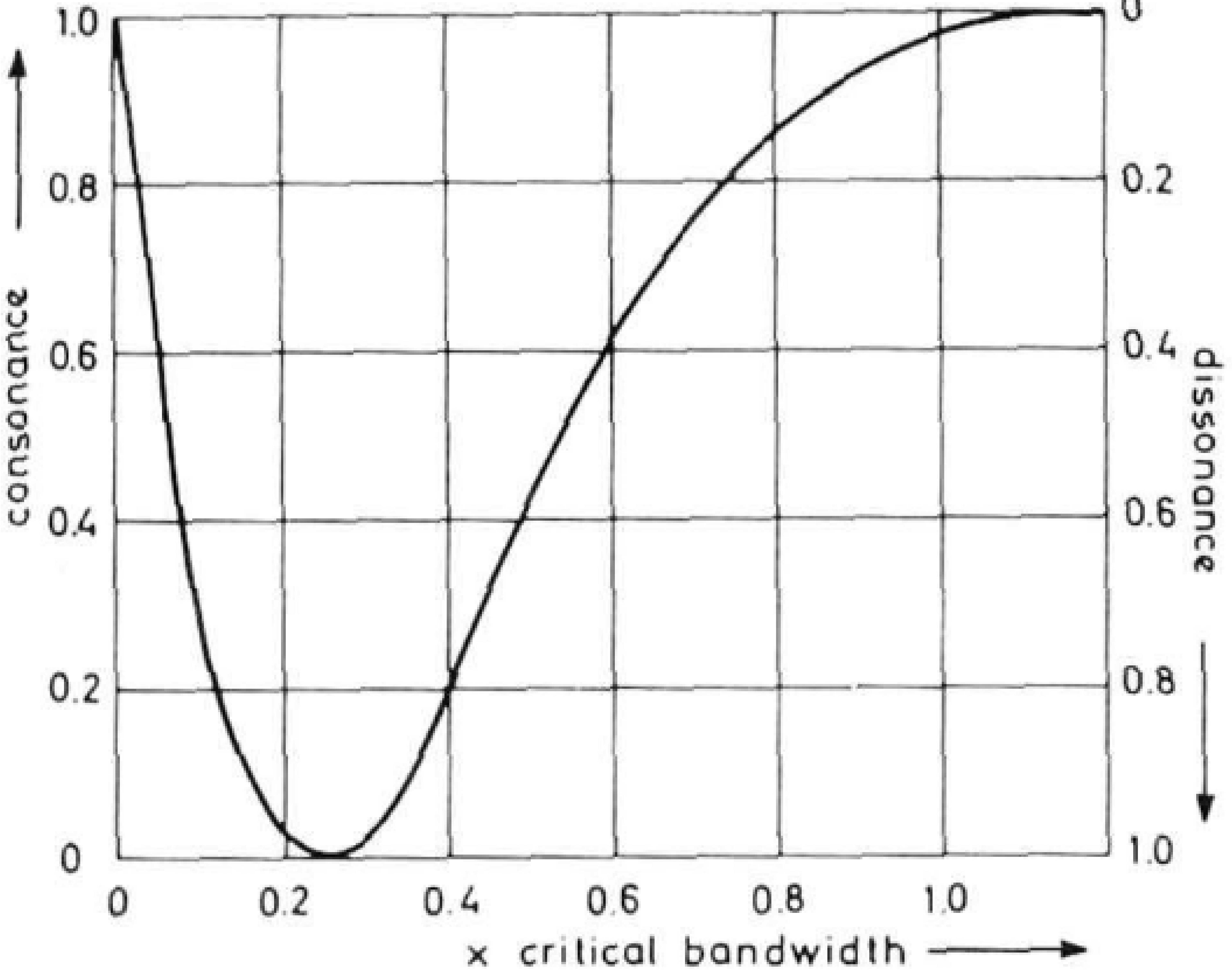



f
$\frac{3}{2}f$

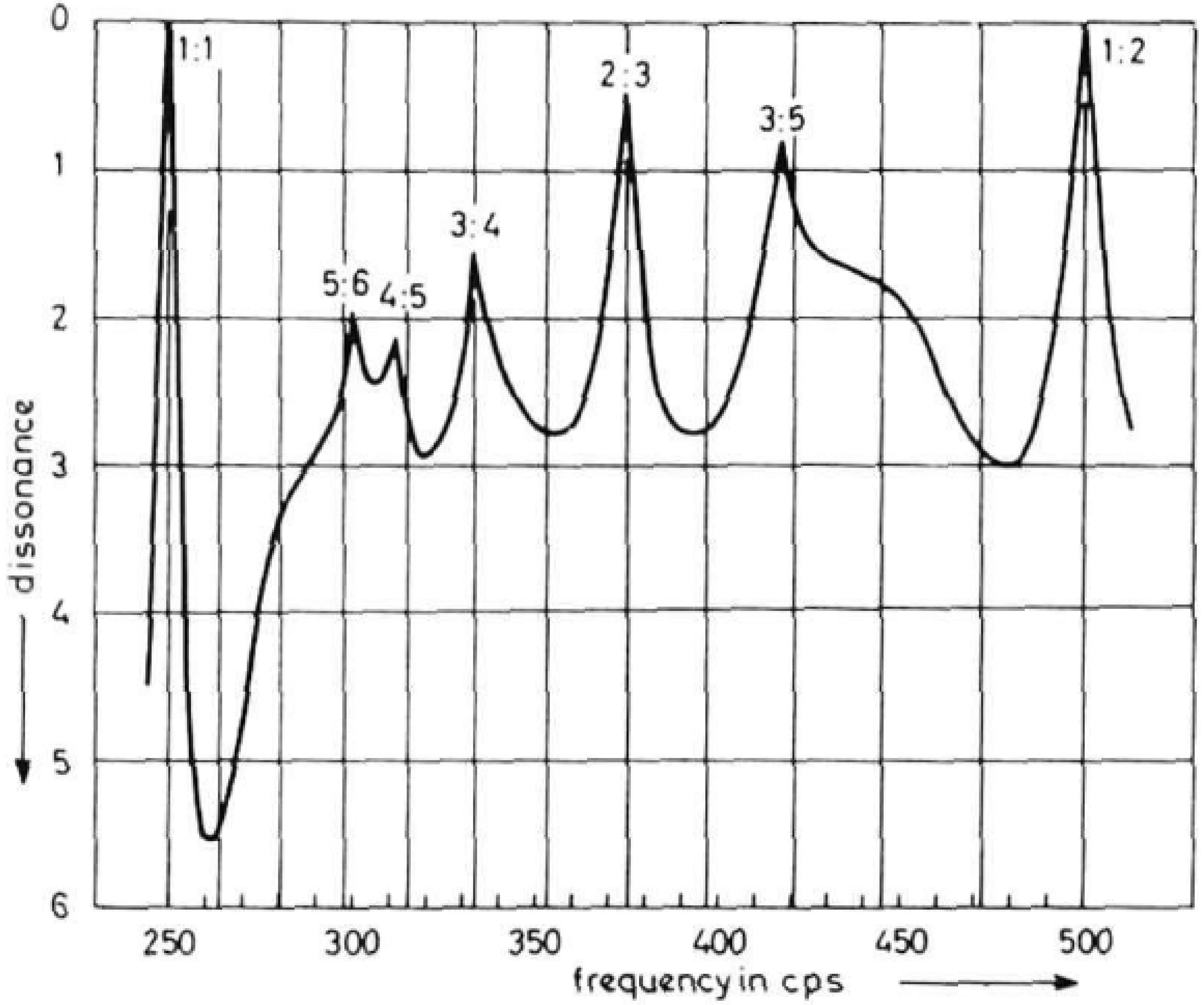

1
0.5
0
−0.5
−1
0
1
2
3
4
5
6
7



1
0.5
0
−0.5
−1
0
1
2
3
4
5
6
7
2
1
0
−1
−2
0
1
2
3
4
5
6
7



William A. Sethares: Tuning, Timbre, Spectrum, Scale



R. Plomp, W. J. M. Levelt: Tonal Consonance and Critical Bandwidth

Principle 6: Western music adds cultural constraints

$$\frac{16}{15}$$

$$\frac{16}{15} = \frac{2.2.2.2}{3.5}$$

$$\frac{16}{15} = \frac{2.2.2.2}{3.5}$$

$$\frac{9}{8} = \frac{3.3}{2.2.2}$$

$$\frac{16}{15} = \frac{2.2.2.2}{3.5} \qquad \frac{9}{8} = \frac{3.3}{2.2.2}$$

$$\frac{16}{15}.\frac{9}{8} = \frac{2.2.2.2.3.3}{3.5.2.2.2} = \frac{3.2}{5} = \frac{6}{5}$$

$$\frac{16}{15} = \frac{2.2.2.2}{3.5} \qquad \frac{9}{8} = \frac{3.3}{2.2.2}$$

$$\frac{16}{15} \cdot \frac{9}{8} = \frac{2.2.2.2.3.3}{3.5.2.2.2} = \frac{3.2}{5} = \frac{6}{5}$$

$\frac{16}{15}, \frac{3}{2}$ yes

$\frac{7}{6}, \frac{22}{15}$ no

7-tone scale

Principle 7: Human hearing is not perfect

Principle 1: We perceive fundamental frequency as pitch

Principle 2: Human hearing works on a logarithmic scale

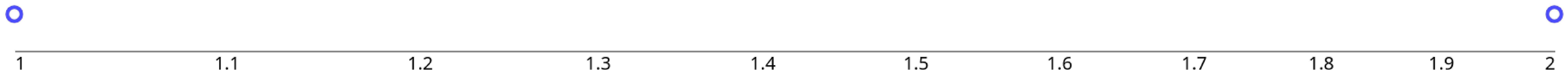
Principle 3: Transpositional invariance

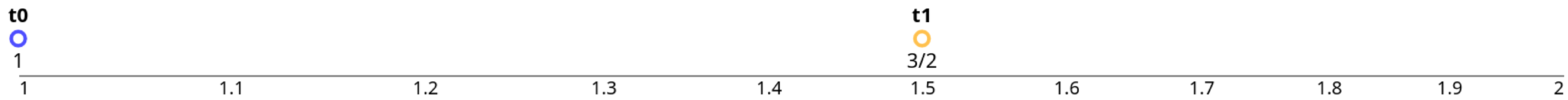
Principle 4: Octave Circularity

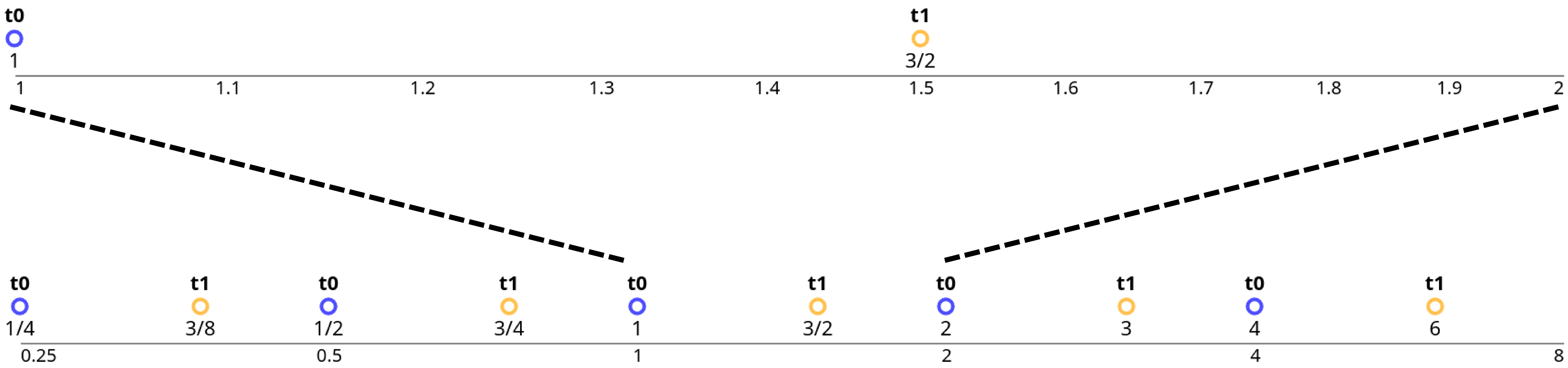
Principle 5: "Small ratios" sound good together

Principle 6: Western music adds cultural constraints

Principle 7: Human hearing is not perfect


1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

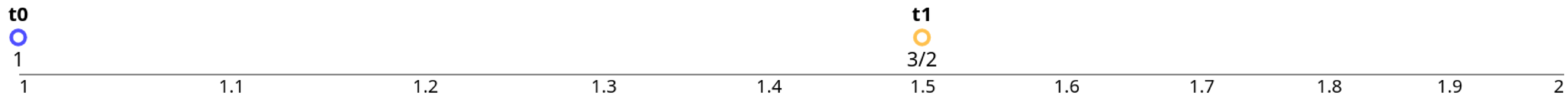

t0
1
t1
3/2
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2



t0
1
t1
3/2
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
t0
1/4
t1
3/8
t0
1/2
t1
3/4
t0
1
t1
3/2
t0
2
t1
3
t0
4
t1
6
0.25
0.5
1
2
4
8

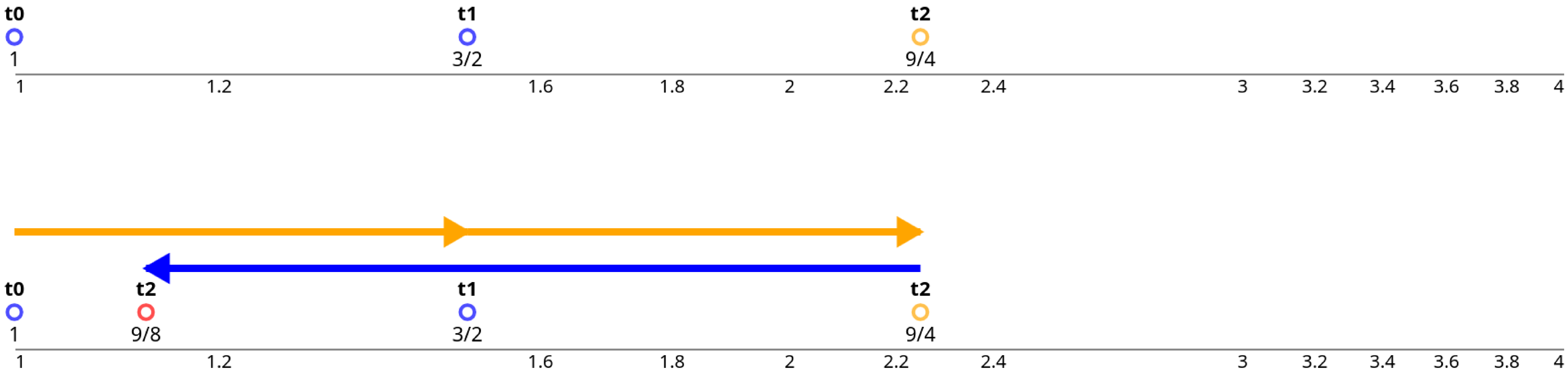
DEMO

3/2 on cello

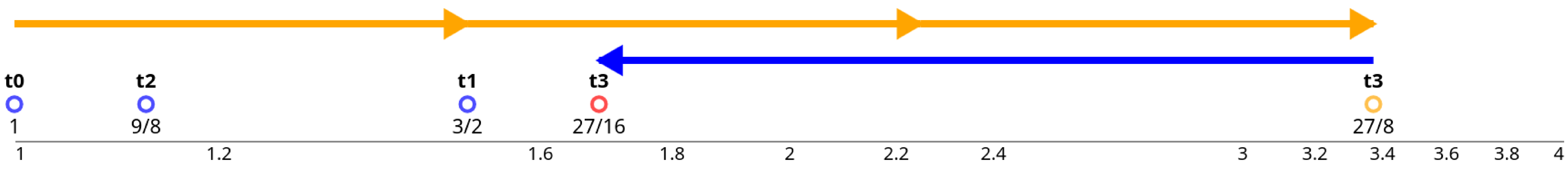

t0
1
t1
3/2
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

$$\frac{3}{2} \cdot \frac{3}{2} = \frac{9}{4}$$

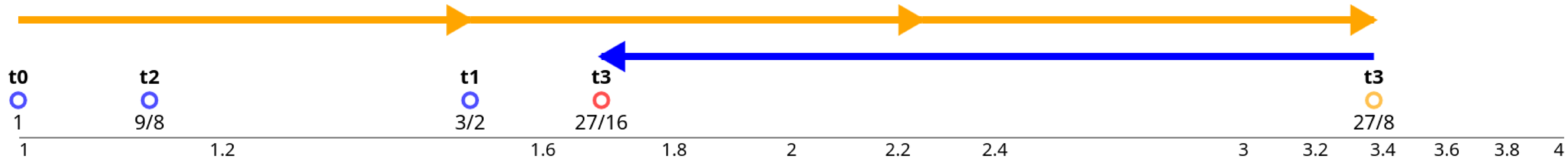
$$\frac{3}{2} \cdot \frac{3}{2} = \frac{9}{4}$$



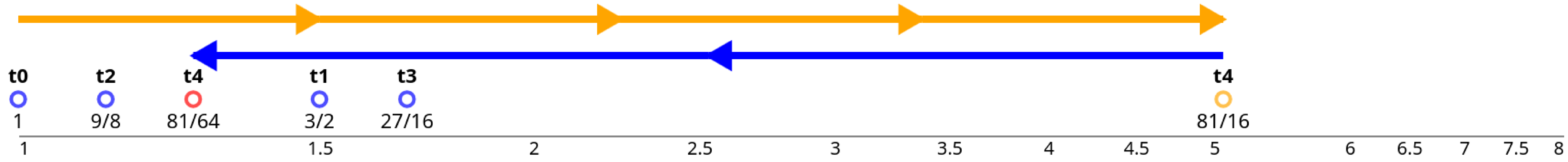
$$\left(\frac{3}{2}\right)^3$$

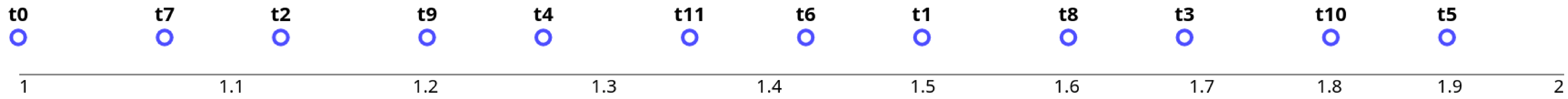


$$\left(\frac{3}{2}\right)^3$$

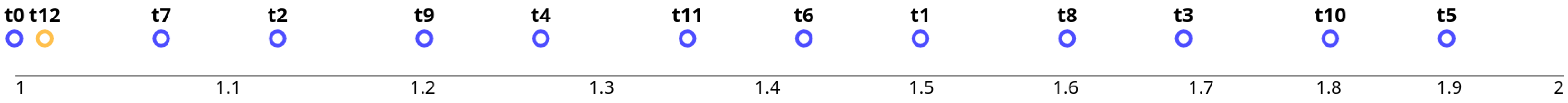


$$\left(\frac{3}{2}\right)^4$$





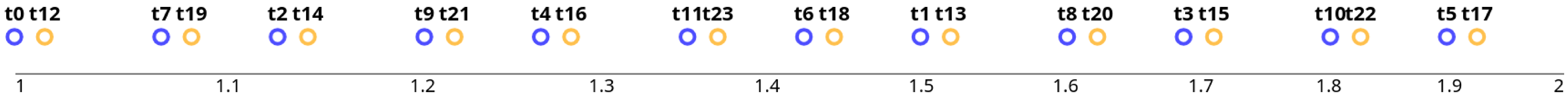
t0
t7
t2
t9
t4
t11
t6
t1
t8
t3
t10
t5
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

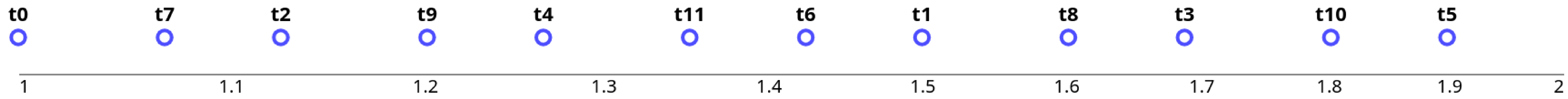

t0
t12
t7
t2
t9
t4
t11
t6
t1
t8
t3
t10
t5
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

$$\left(\frac{3}{2}\right)^{12} = \frac{531441}{4096} \sim 129.7463$$

$$\left(\frac{3}{2}\right)^{12} = \frac{531441}{4096} \sim 129.7463$$

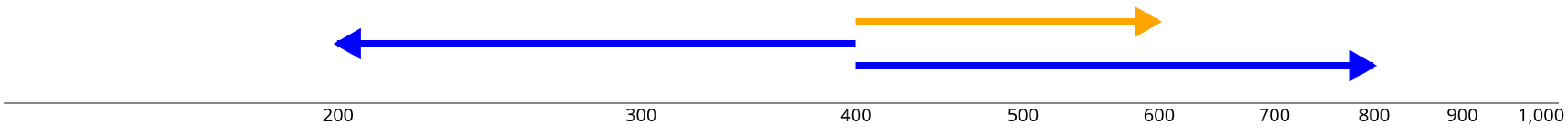
$$128 = 2^7$$

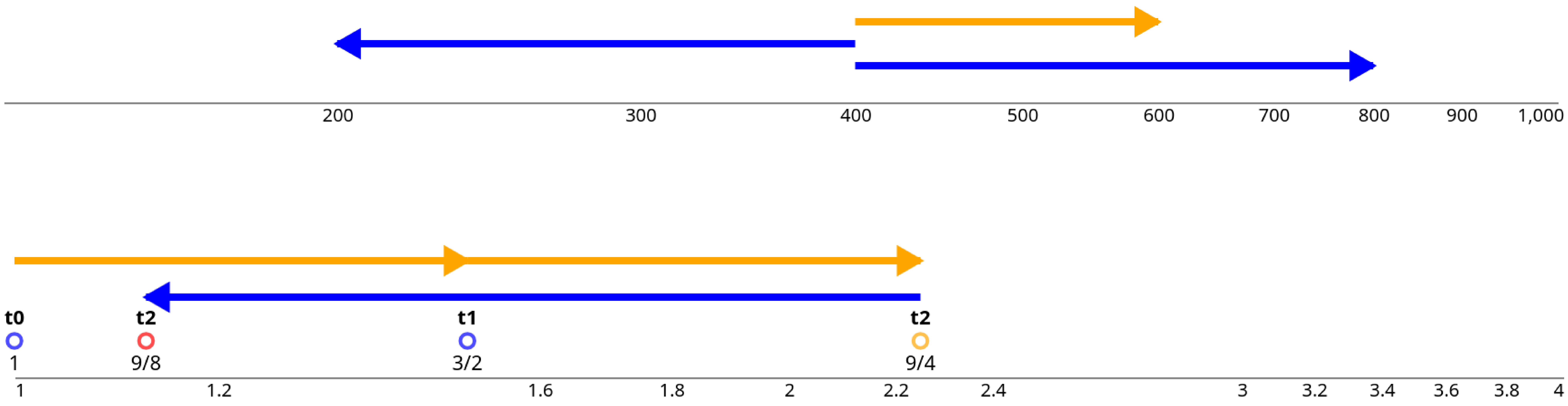

t0 t12
t7 t19
t2 t14
t9 t21
t4 t16
t11t23
t6 t18
t1 t13
t8 t20
t3 t15
t10t22
t5 t17
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2


t0
t7
t2
t9
t4
t11
t6
t1
t8
t3
t10
t5
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

DEMO

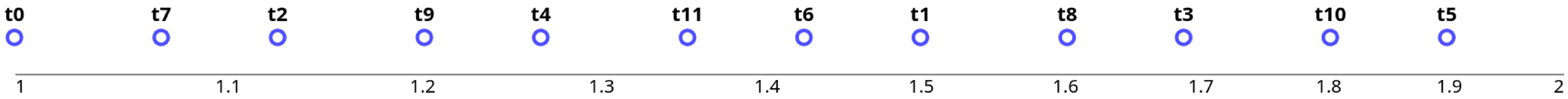
puleni struny


200
300
400
500
600
700
800
900
1,000

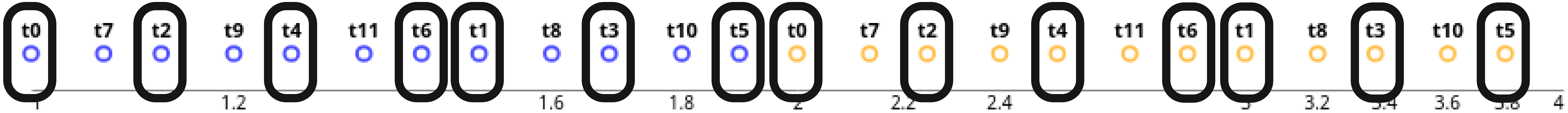

200
300
400
500
600
700
800
900
1,000
t0
t2
t1
t2
1
9/8
3/2
9/4
1
1.2
1.6
1.8
2
2.2
2.4
3
3.2
3.4
3.6
3.8
4

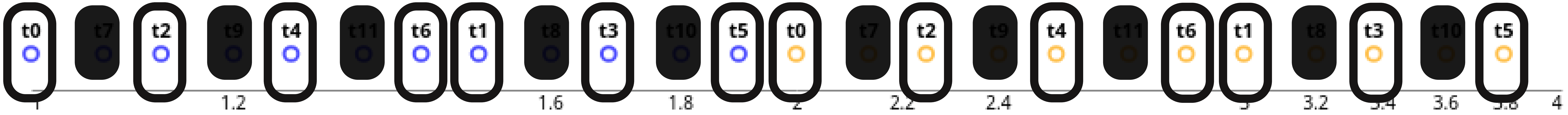
$$t_0 = 1, t_1 = \frac{3}{2}, t_2 = \frac{9}{8}, t_3 = \frac{27}{16}, t_4 = \frac{81}{64}, t_5 = \frac{243}{128}, t_6 = \frac{729}{512}$$

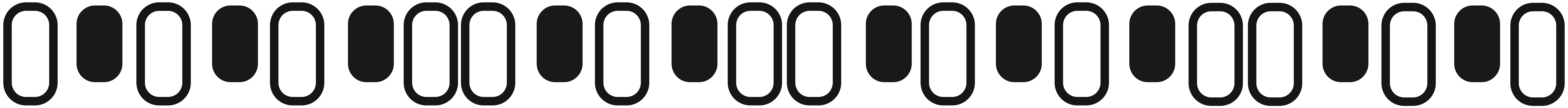
$$t_7 = \frac{2187}{2048}, t_8 = \frac{6561}{4096}, t_9 = \frac{19683}{16384}, t_{10} = \frac{59049}{32768}, t_{11} = \frac{177147}{131072}$$

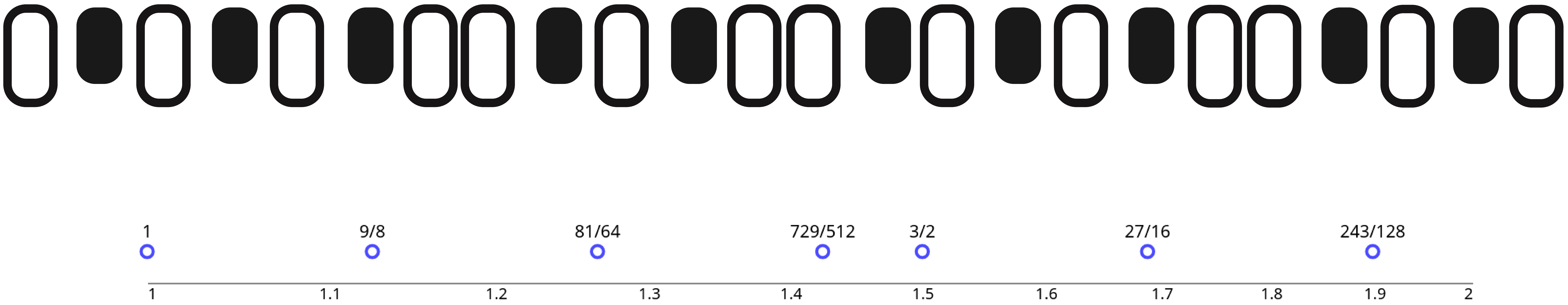

t0
t7
t2
t9
t4
t11
t6
t1
t8
t3
t10
t5
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

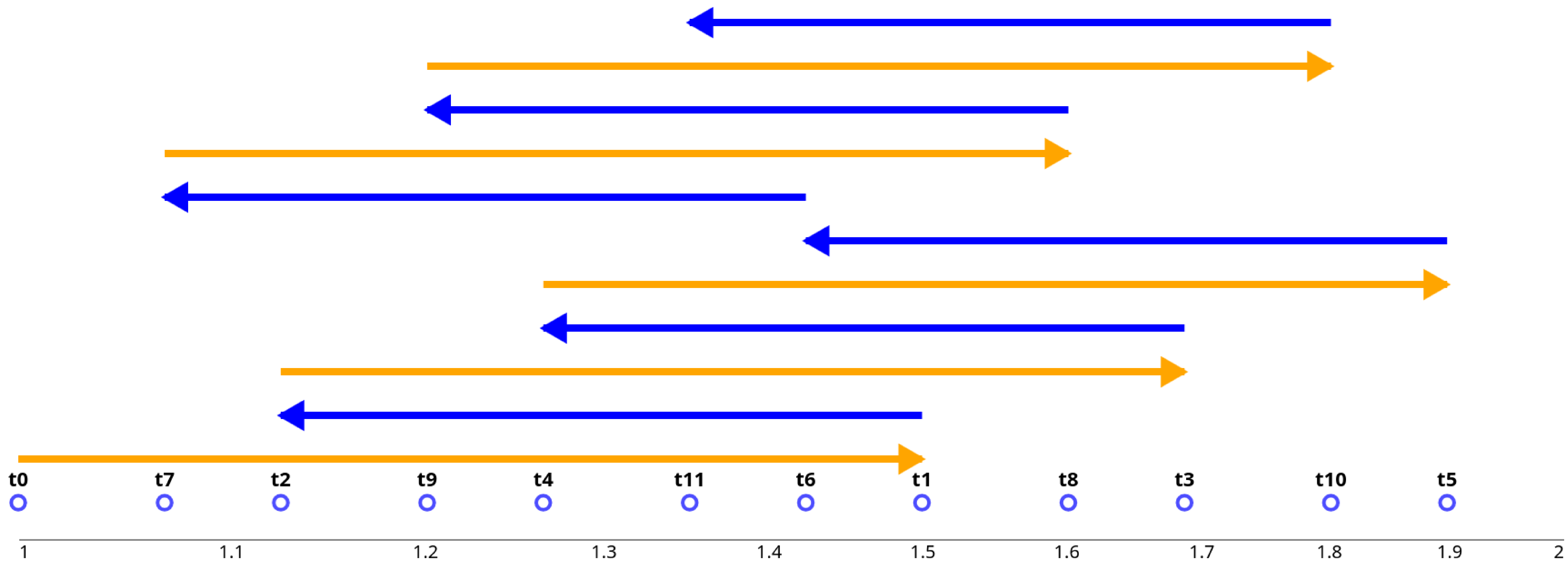

t0
t7
t2
t9
t4
t11
t6
t1
t8
t3
t10
t5
t0
t7
t2
t9
t4
t11
t6
t1
t8
t3
t10
t5
1
1.2
1.6
1.8
2
2.2
2.4
3
3.2
3.4
3.6
3.8
4


t0 t7 t2 t9 t4 t11 t6 t1 t8 t3 t10 t5 t0 t7 t2 t9 t4 t11 t6 t1 t8 t3 t10 t5
1.2
1.6
1.8
2.4
3.2
3.6
4


t0
t2
t4
t6
t1
t3
t5
t0
t2
t4
t6
t1
t3
t5
1.2
1.6
1.8
2.2
2.4
3.2
3.6
4




1
9/8
81/64
729/512
3/2
27/16
243/128
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2


t0
t7
t2
t9
t4
t11
t6
t1
t8
t3
t10
t5
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

$$\frac{1}{1} = 1.0$$

$$\frac{9}{8} = 1.125$$

$$\frac{81}{64} \sim 1.266$$

$$\frac{729}{512} \sim 1.424$$

$$\frac{3}{2} = 1.5$$

$$\frac{27}{16} \sim 1.688$$

$$\frac{243}{128} \sim 1.898$$

$\frac{1}{1} = 1.0$ $\longrightarrow$ $\frac{1}{1} = 1.0$

$\frac{9}{8} = 1.125$ $\longrightarrow$ $\frac{9}{8} = 1.125$

$\frac{81}{64} \sim 1.266$

$\frac{729}{512} \sim 1.424$

$\frac{3}{2} = 1.5$ $\longrightarrow$ $\frac{3}{2} = 1.5$

$\frac{27}{16} \sim 1.688$

$\frac{243}{128} \sim 1.898$

$\frac{1}{1} = 1.0$	$\longrightarrow$	$\frac{1}{1} = 1.0$
$\frac{9}{8} = 1.125$	$\longrightarrow$	$\frac{9}{8} = 1.125$
$\frac{81}{64} \sim 1.266$		
$\frac{729}{512} \sim 1.424$	$\longrightarrow$	$\frac{4}{3} \sim 1.333$
$\frac{3}{2} = 1.5$	$\longrightarrow$	$\frac{3}{2} = 1.5$
$\frac{27}{16} \sim 1.688$		
$\frac{243}{128} \sim 1.898$		

$\frac{1}{1} = 1.0 \longrightarrow \frac{1}{1} = 1.0$

$\frac{9}{8} = 1.125 \longrightarrow \frac{9}{8} = 1.125$

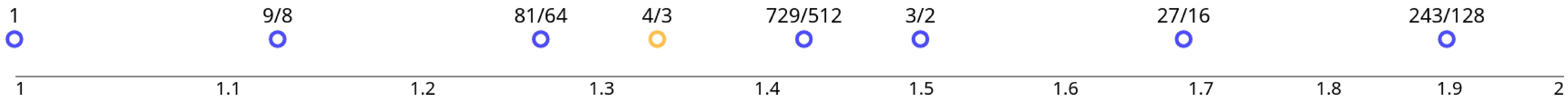
$\frac{81}{64} \sim 1.266$

$\frac{729}{512} \sim 1.424 \longrightarrow \frac{4}{3} \sim 1.333$

$\frac{3}{2} = 1.5 \longrightarrow \frac{3}{2} = 1.5$

$\frac{27}{16} \sim 1.688$

$\frac{243}{128} \sim 1.898$



$\frac{1}{1} = 1.0 \rightarrow \frac{1}{1} = 1.0$

$\frac{9}{8} = 1.125 \rightarrow \frac{9}{8} = 1.125$

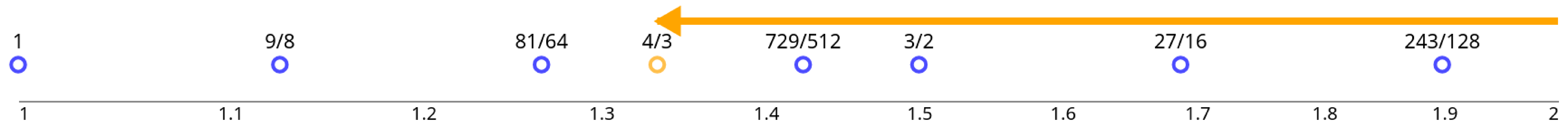
$\frac{81}{64} \sim 1.266$

$\frac{729}{512} \sim 1.424 \rightarrow \frac{4}{3} \sim 1.333$

$\frac{3}{2} = 1.5 \rightarrow \frac{3}{2} = 1.5$

$\frac{27}{16} \sim 1.688$

$\frac{243}{128} \sim 1.898$



$\frac{1}{1} = 1.0 \longrightarrow \frac{1}{1} = 1.0$

$\frac{9}{8} = 1.125 \longrightarrow \frac{9}{8} = 1.125$

$\frac{81}{64} \sim 1.266 \longrightarrow \frac{5}{4} = 1.25$

$\frac{729}{512} \sim 1.424 \longrightarrow \frac{4}{3} \sim 1.333$

$\frac{3}{2} = 1.5 \longrightarrow \frac{3}{2} = 1.5$

$\frac{27}{16} \sim 1.688$

$\frac{243}{128} \sim 1.898$

$\frac{1}{1} = 1.0 \longrightarrow \frac{1}{1} = 1.0$

$\frac{9}{8} = 1.125 \longrightarrow \frac{9}{8} = 1.125$

$\frac{81}{64} \sim 1.266 \longrightarrow \frac{5}{4} = 1.25$

$\frac{729}{512} \sim 1.424 \longrightarrow \frac{4}{3} \sim 1.333$

$\frac{3}{2} = 1.5 \longrightarrow \frac{3}{2} = 1.5$

$\frac{27}{16} \sim 1.688 \longrightarrow \frac{5}{3} \sim 1.667$

$\frac{243}{128} \sim 1.898$

$\frac{1}{1} = 1.0 \longrightarrow \frac{1}{1} = 1.0$

$\frac{9}{8} = 1.125 \longrightarrow \frac{9}{8} = 1.125$

$\frac{81}{64} \sim 1.266 \longrightarrow \frac{5}{4} = 1.25$

$\frac{729}{512} \sim 1.424 \longrightarrow \frac{4}{3} \sim 1.333$

$\frac{3}{2} = 1.5 \longrightarrow \frac{3}{2} = 1.5$

$\frac{27}{16} \sim 1.688 \longrightarrow \frac{5}{3} \sim 1.667$

$\frac{243}{128} \sim 1.898 \longrightarrow \frac{15}{8} = 1.875$

Claudius Ptolemy

$\frac{1}{1} \sim 1.0 \longrightarrow \frac{1}{1} \sim 1.0$

$\frac{9}{8} \sim 1.125 \longrightarrow \frac{9}{8} \sim 1.125$

$\frac{81}{64} \sim 1.266 \longrightarrow \frac{5}{4} \sim 1.25$

$\frac{729}{512} \sim 1.424 \longrightarrow \frac{4}{3} \sim 1.333$

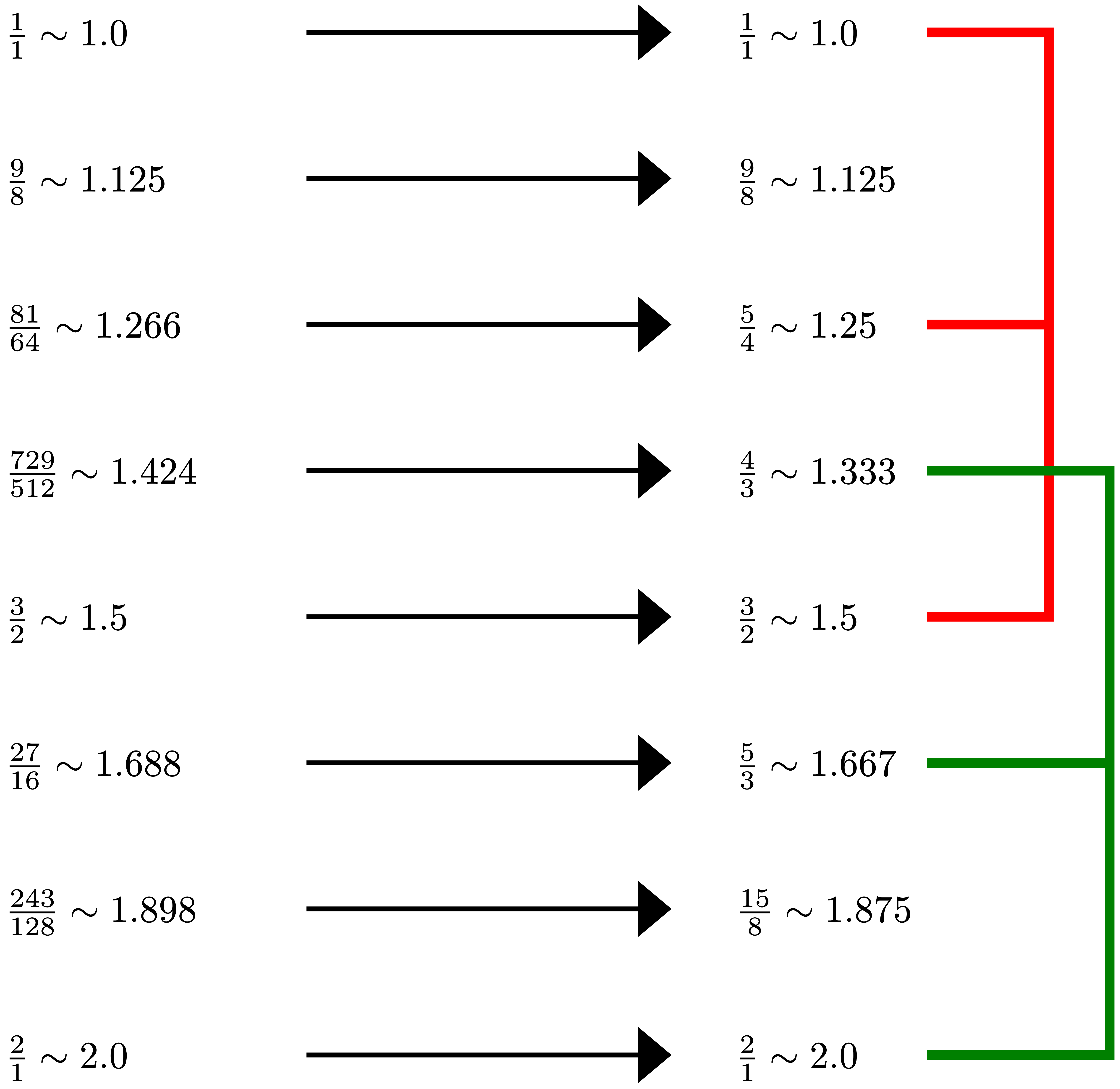
$\frac{3}{2} \sim 1.5 \longrightarrow \frac{3}{2} \sim 1.5$

$\frac{27}{16} \sim 1.688 \longrightarrow \frac{5}{3} \sim 1.667$

$\frac{243}{128} \sim 1.898 \longrightarrow \frac{15}{8} \sim 1.875$

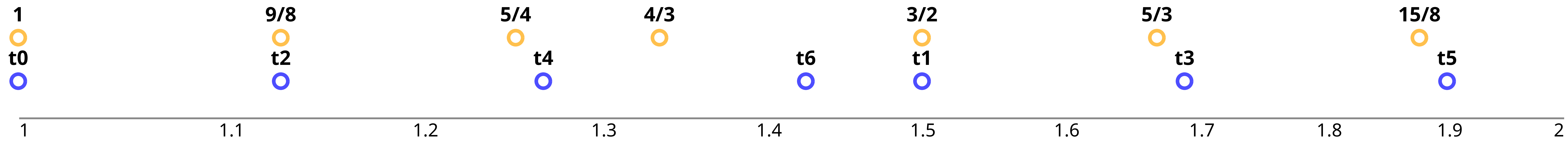
$\frac{2}{1} \sim 2.0 \longrightarrow \frac{2}{1} \sim 2.0$

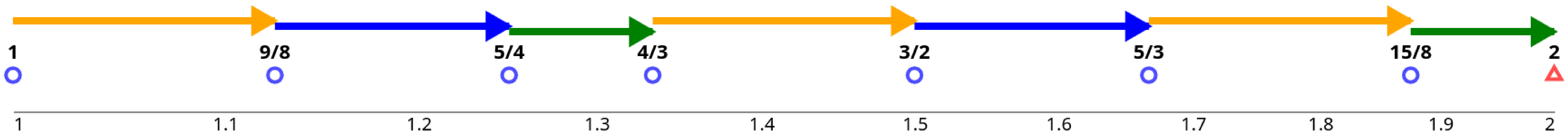
$\frac{1}{1} \sim 1.0$	$\longrightarrow$	$\frac{1}{1} \sim 1.0$
$\frac{9}{8} \sim 1.125$	$\longrightarrow$	$\frac{9}{8} \sim 1.125$
$\frac{81}{64} \sim 1.266$	$\longrightarrow$	$\frac{5}{4} \sim 1.25$
$\frac{729}{512} \sim 1.424$	$\longrightarrow$	$\frac{4}{3} \sim 1.333$
$\frac{3}{2} \sim 1.5$	$\longrightarrow$	$\frac{3}{2} \sim 1.5$
$\frac{27}{16} \sim 1.688$	$\longrightarrow$	$\frac{5}{3} \sim 1.667$
$\frac{243}{128} \sim 1.898$	$\longrightarrow$	$\frac{15}{8} \sim 1.875$
$\frac{2}{1} \sim 2.0$	$\longrightarrow$	$\frac{2}{1} \sim 2.0$



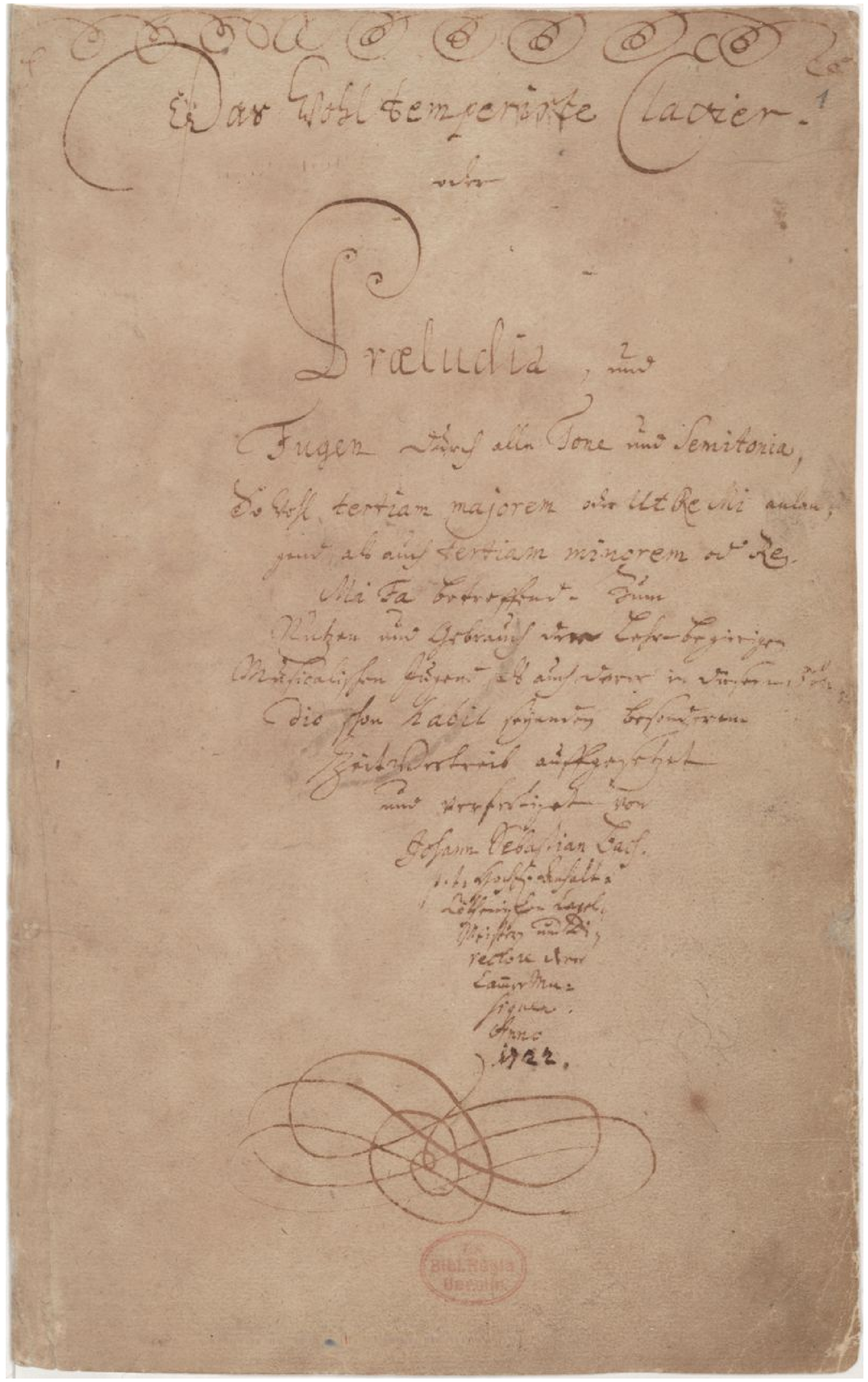
$\frac{1}{1} \sim 1.0$
$\frac{9}{8} \sim 1.125$
$\frac{81}{64} \sim 1.266$
$\frac{729}{512} \sim 1.424$
$\frac{3}{2} \sim 1.5$
$\frac{27}{16} \sim 1.688$
$\frac{243}{128} \sim 1.898$
$\frac{2}{1} \sim 2.0$
$\frac{1}{1} \sim 1.0$
$\frac{9}{8} \sim 1.125$
$\frac{5}{4} \sim 1.25$
$\frac{4}{3} \sim 1.333$
$\frac{3}{2} \sim 1.5$
$\frac{5}{3} \sim 1.667$
$\frac{15}{8} \sim 1.875$
$\frac{2}{1} \sim 2.0$

	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
1	1	$\frac{8}{9}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{8}{15}$	$\frac{1}{2}$
$\frac{9}{8}$	$\frac{9}{8}$	1	$\frac{9}{10}$	$\frac{27}{32}$	$\frac{3}{4}$	$\frac{27}{40}$	$\frac{3}{5}$	$\frac{9}{16}$
$\frac{5}{4}$	$\frac{5}{4}$	$\frac{10}{9}$	1	$\frac{15}{16}$	$\frac{5}{6}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{5}{8}$
$\frac{4}{3}$	$\frac{4}{3}$	$\frac{32}{27}$	$\frac{16}{15}$	1	$\frac{8}{9}$	$\frac{4}{5}$	$\frac{32}{45}$	$\frac{2}{3}$
$\frac{3}{2}$	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{6}{5}$	$\frac{9}{8}$	1	$\frac{9}{10}$	$\frac{4}{5}$	$\frac{3}{4}$
$\frac{5}{3}$	$\frac{5}{3}$	$\frac{40}{27}$	$\frac{4}{3}$	$\frac{5}{4}$	$\frac{10}{9}$	1	$\frac{8}{9}$	$\frac{5}{6}$
$\frac{15}{8}$	$\frac{15}{8}$	$\frac{5}{3}$	$\frac{3}{2}$	$\frac{45}{32}$	$\frac{5}{4}$	$\frac{9}{8}$	1	$\frac{15}{16}$
2	2	$\frac{16}{9}$	$\frac{8}{5}$	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{6}{5}$	$\frac{16}{15}$	1

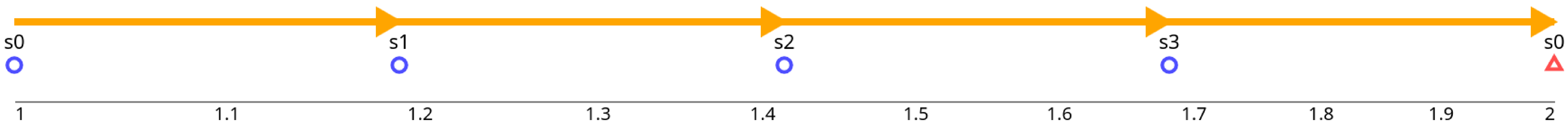

1
9/8
5/4
4/3
3/2
5/3
15/8
t0
t2
t4
t6
t1
t3
t5
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

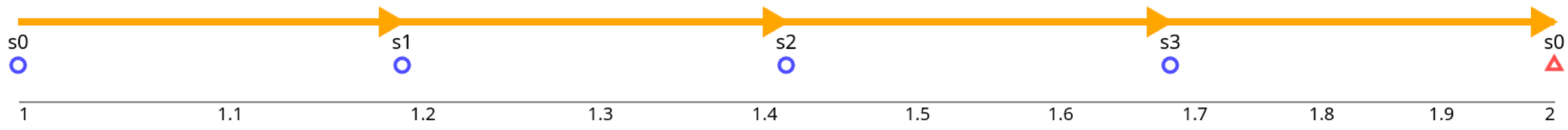


$$\frac{9}{8} \qquad \frac{10}{9} \qquad \frac{16}{15}$$

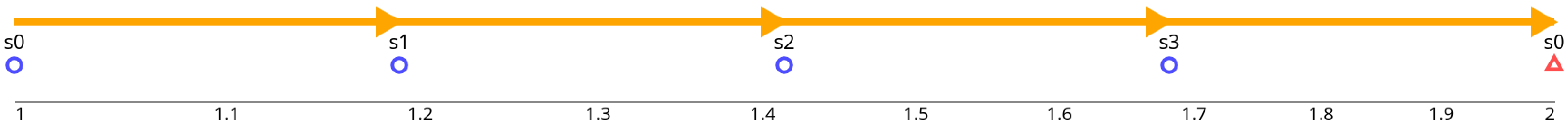


Johann Sebastian Bach: **The Well-Tempered Clavier**


s0
s1
s2
s3
s0
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

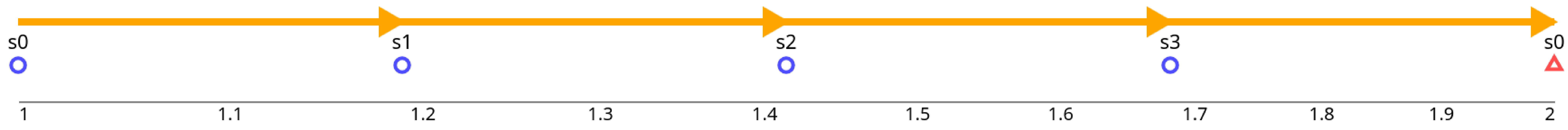


$$c.c.c.c = 2$$



$$c.c.c.c = 2$$

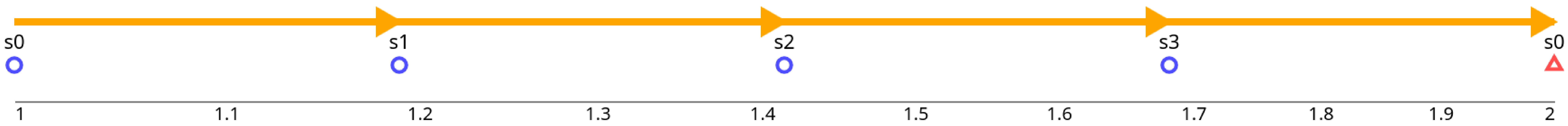
$$c^4 = 2$$



$$c.c.c.c = 2$$

$$c^4 = 2$$

$$c = \sqrt[4]{2}$$

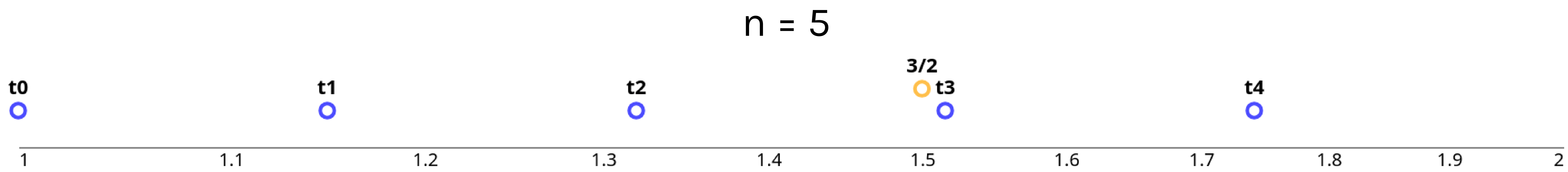


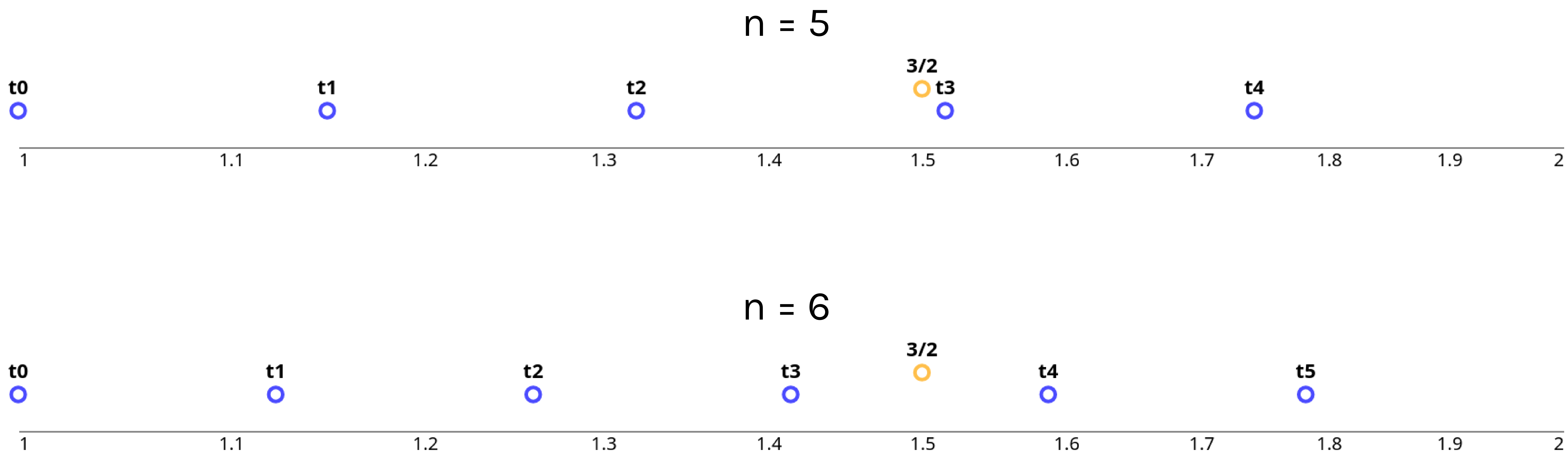
$$c.c.c.c = 2$$

$$c^4 = 2$$

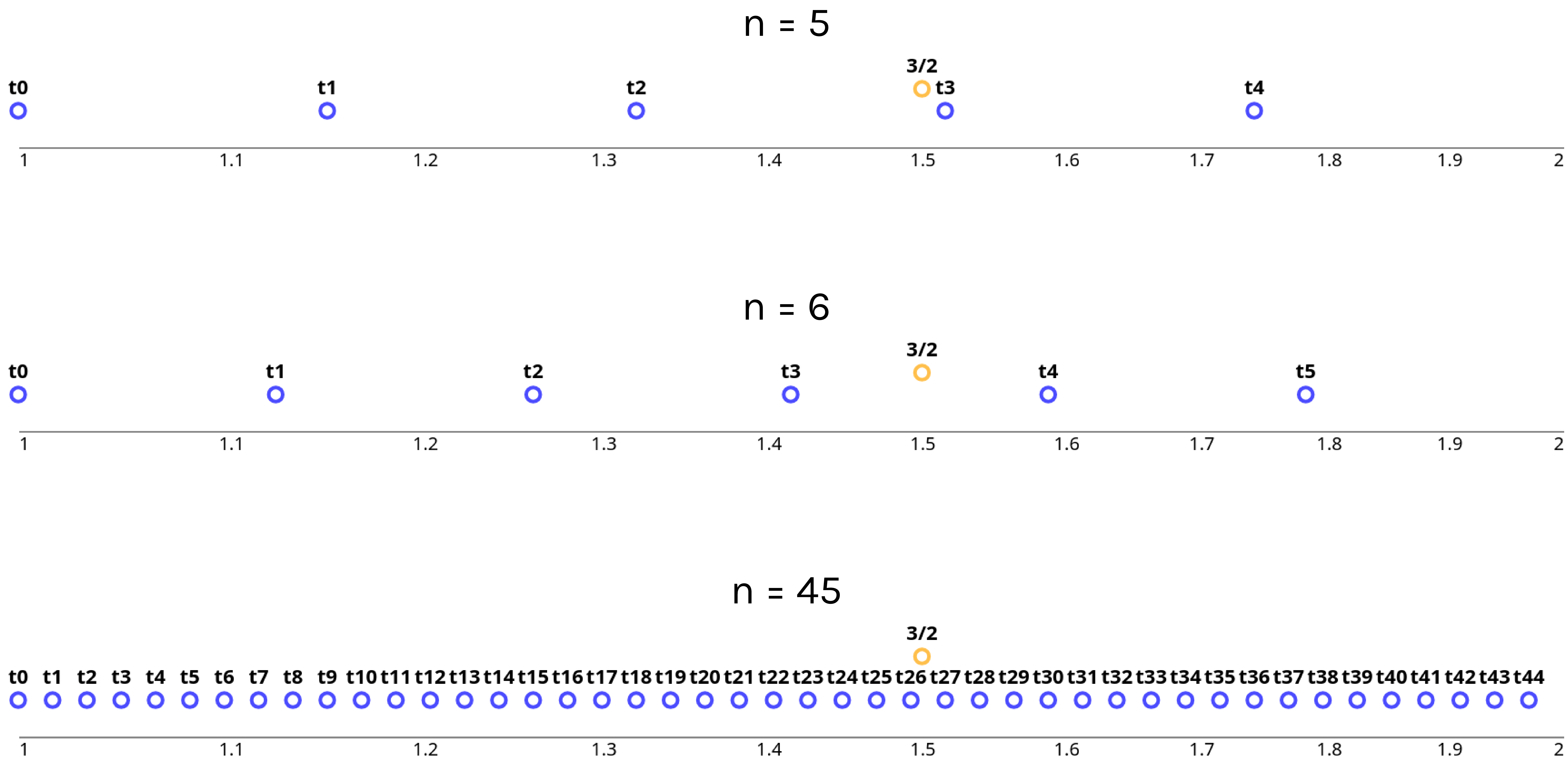
$$c = \sqrt[4]{2}$$

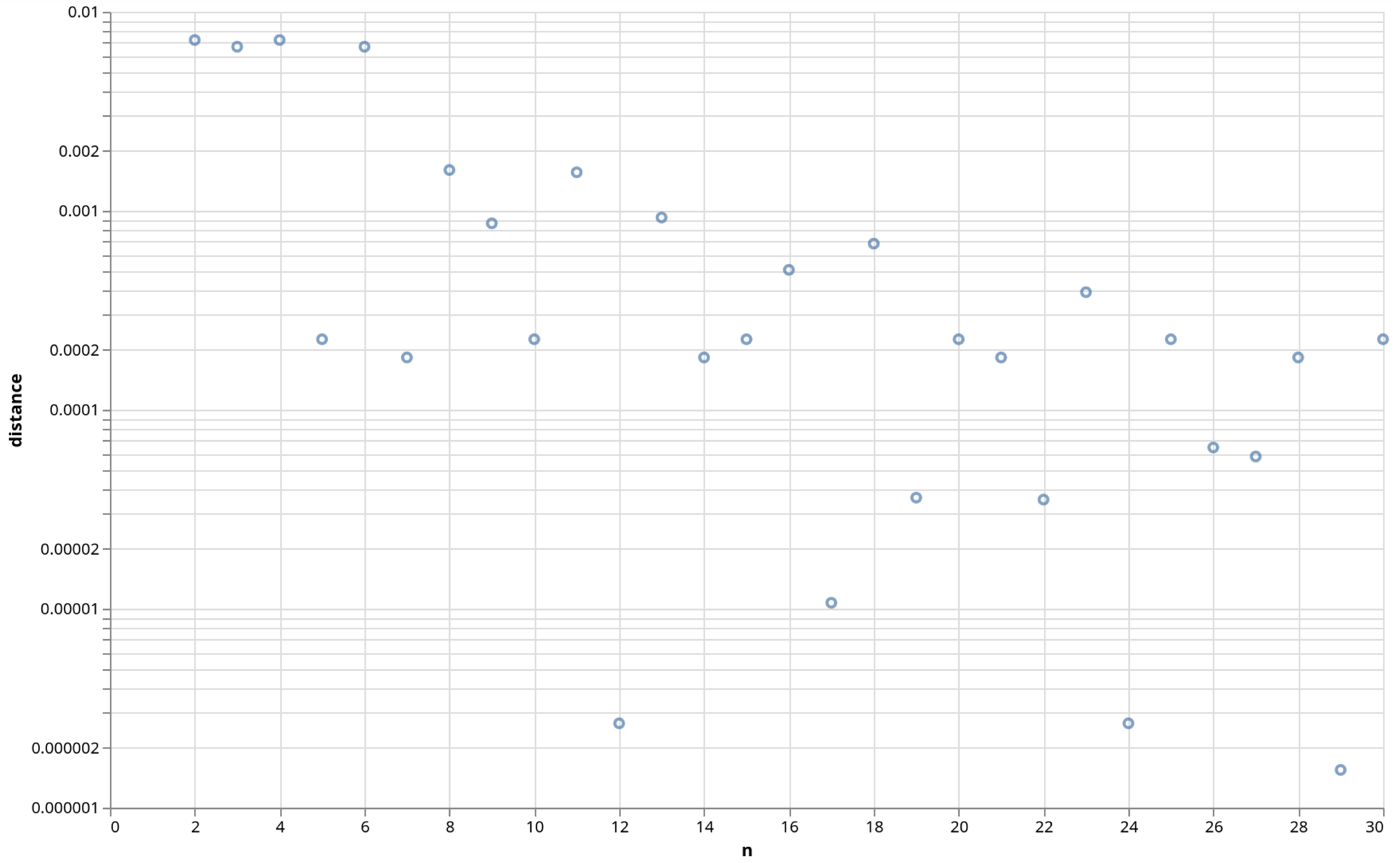
$$c = \sqrt[n]{2}$$

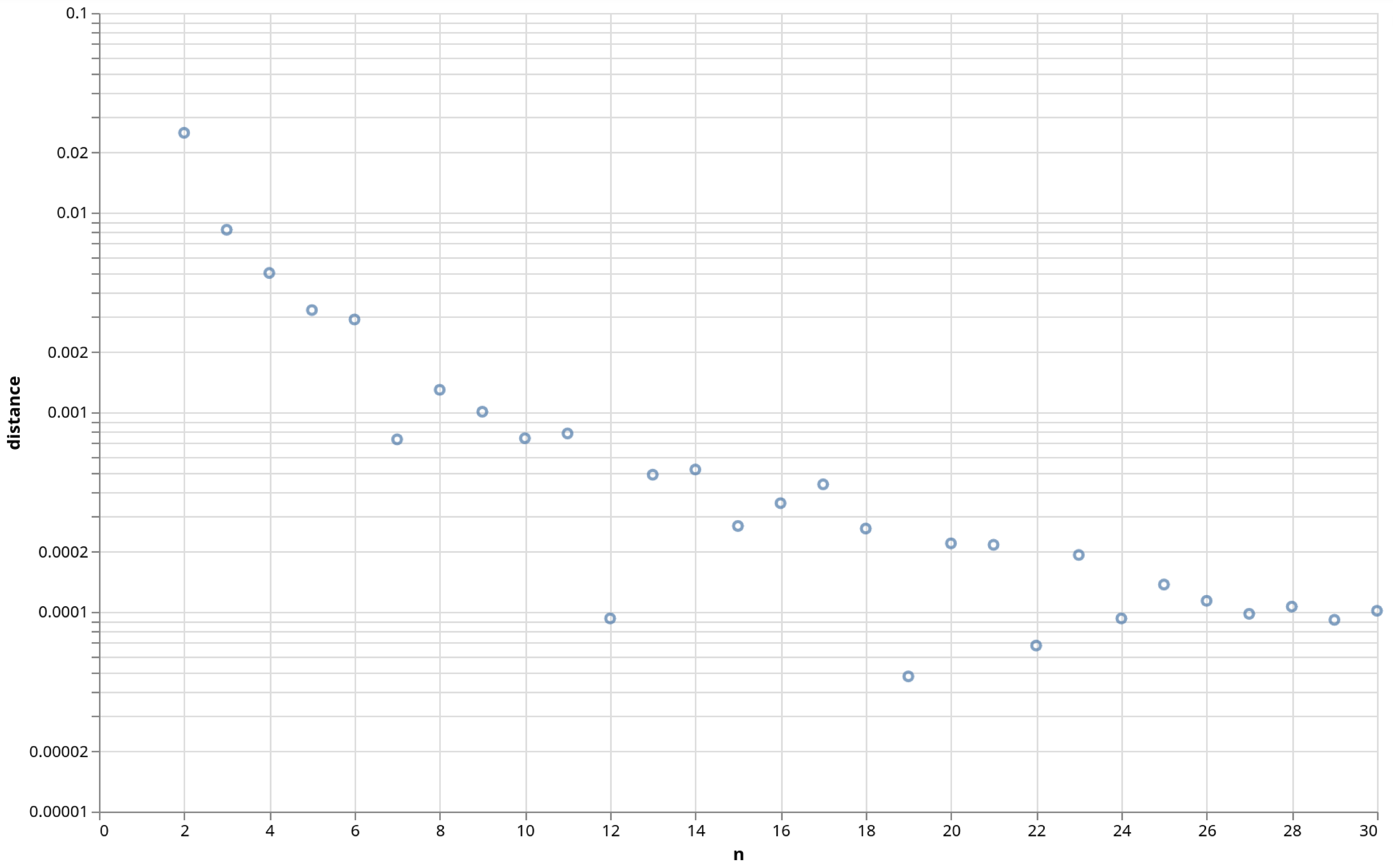

n = 5
t0
t1
t2
3/2
t3
t4
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2



n = 5
t0
t1
t2
3/2
t3
t4
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
n = 6
t0
t1
t2
t3
3/2
t4
t5
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2


n = 5
t0
t1
t2
3/2
t3
t4
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
n = 6
t0
t1
t2
t3
3/2
t4
t5
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
n = 45
3/2
t0 t1 t2 t3 t4 t5 t6 t7 t8 t9 t10 t11 t12 t13 t14 t15 t16 t17 t18 t19 t20 t21 t22 t23 t24 t25 t26 t27 t28 t29 t30 t31 t32 t33 t34 t35 t36 t37 t38 t39 t40 t41 t42 t43 t44
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

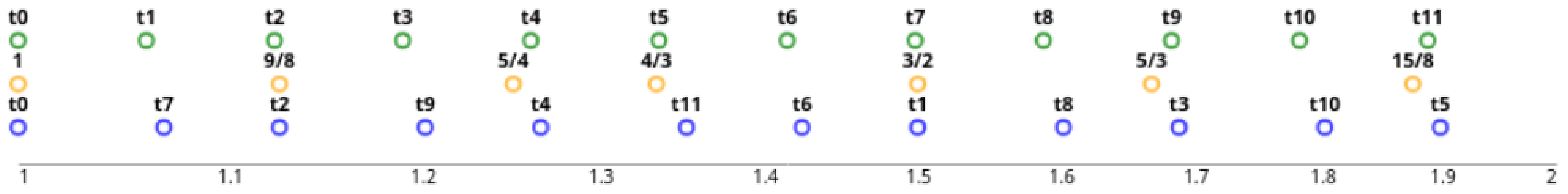

0.01
0.002
0.001
0.0002
0.0001
0.00002
0.00001
0.000002
0.000001
distance
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
n

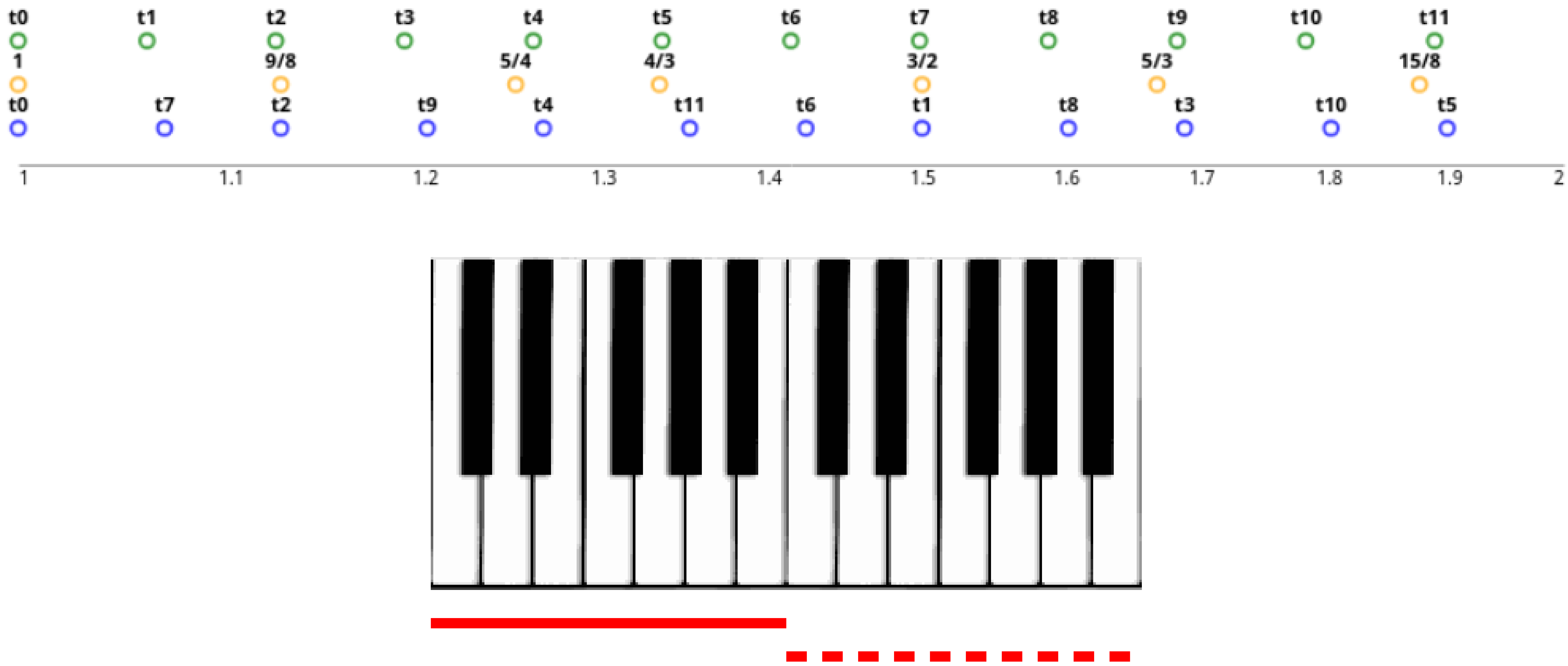


0.1
0.02
0.01
0.002
0.001
0.0002
0.0001
0.00002
0.00001
distance
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
n

$$\sqrt[12]{2} \stackrel{?}{=} \frac{a}{b}$$

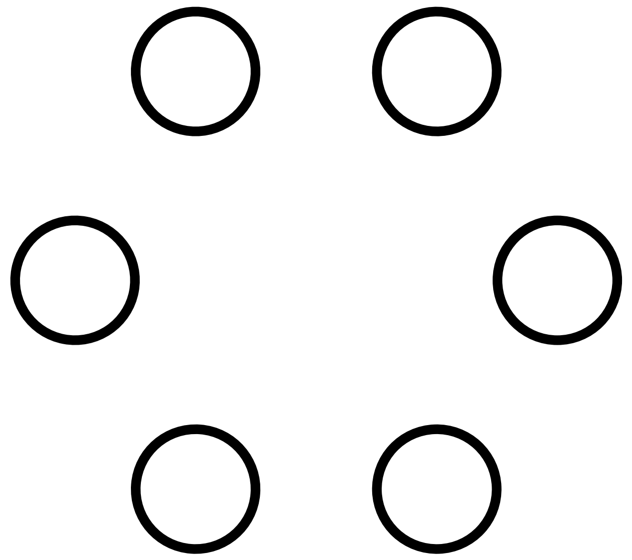
$$\sqrt[12]{2} \neq \frac{a}{b}$$


t0
t1
t2
t3
t4
t5
t6
t7
t8
t9
t10
t11
1
9/8
5/4
4/3
3/2
5/3
15/8
t0
t7
t2
t9
t4
t11
t6
t1
t8
t3
t10
t5
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2


t0 t1 t2 t3 t4 t5 t6 t7 t8 t9 t10 t11
1 9/8 5/4 4/3 3/2 5/3 15/8
t0 t7 t2 t9 t4 t11 t6 t1 t8 t3 t10 t5
1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2

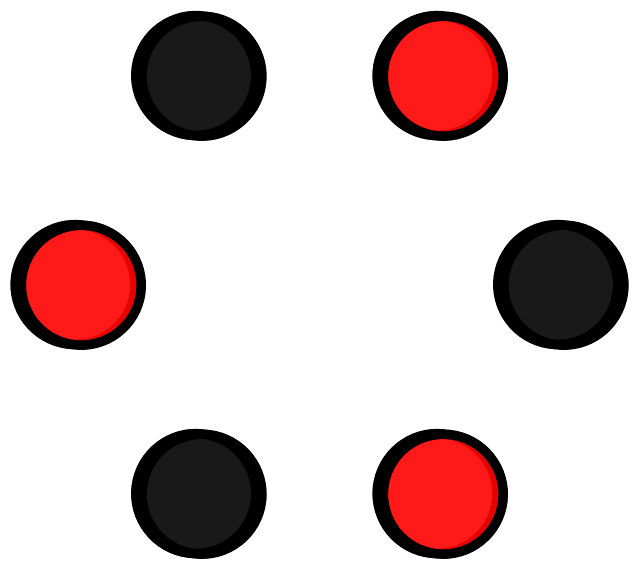
3

3

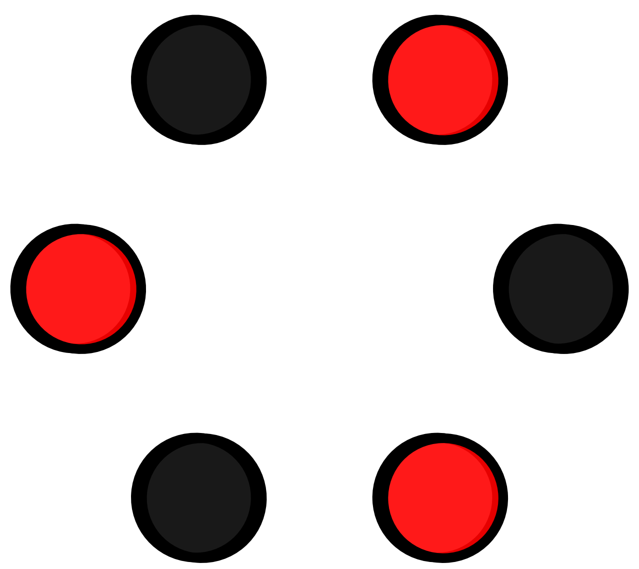


3

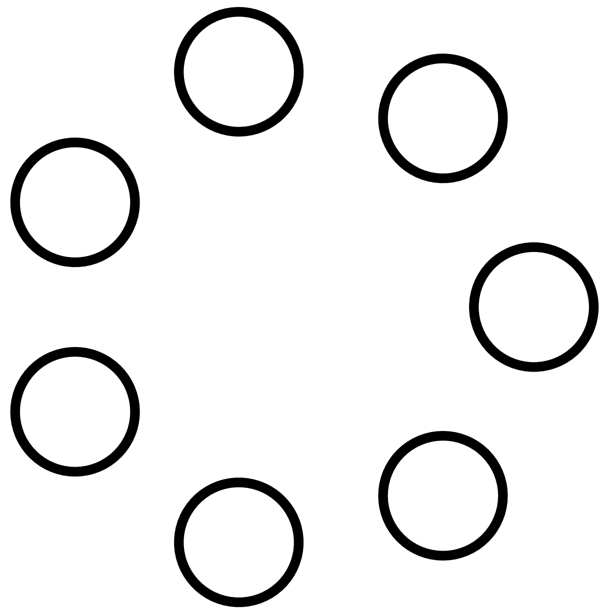
3



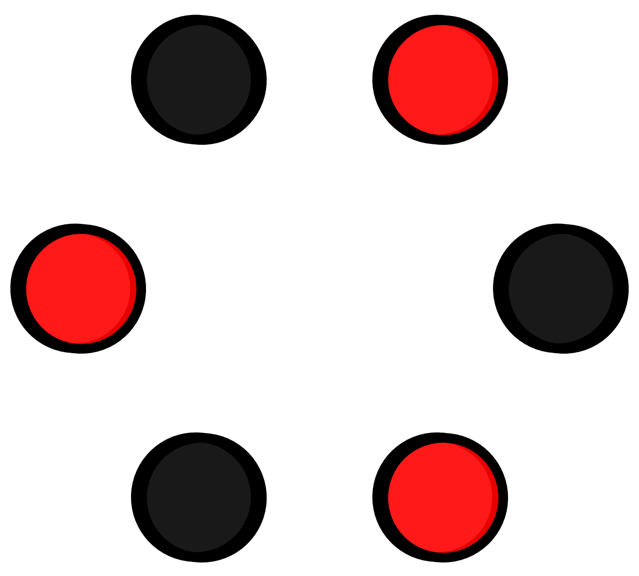
3
3



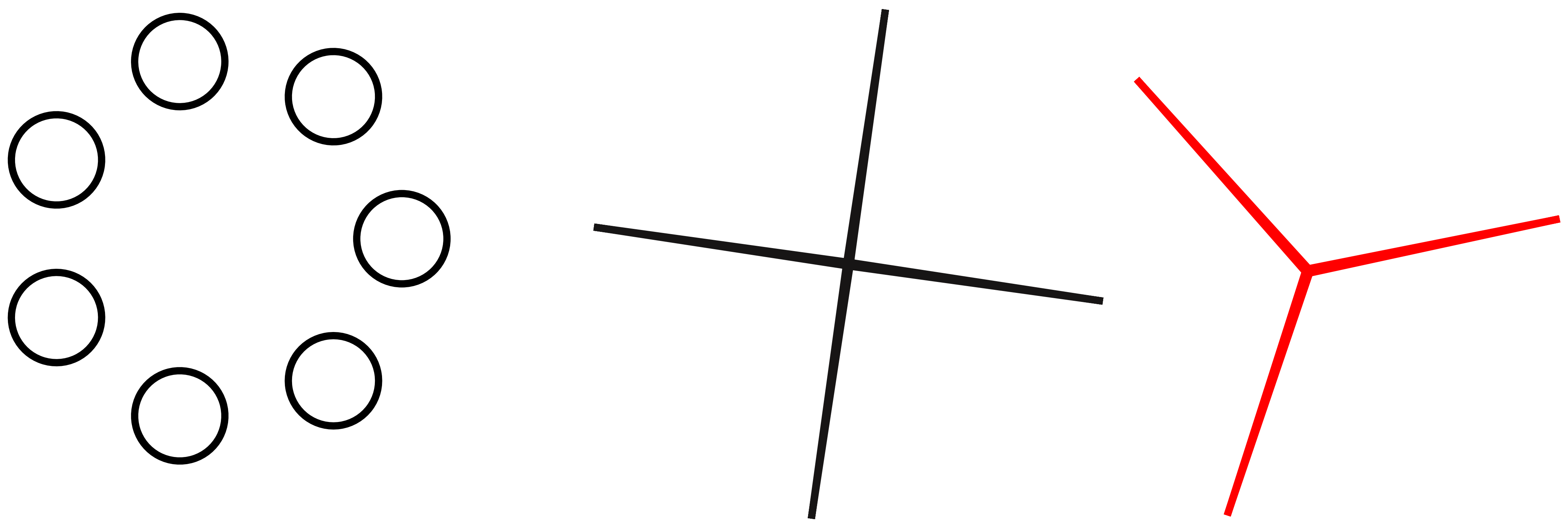
4
3



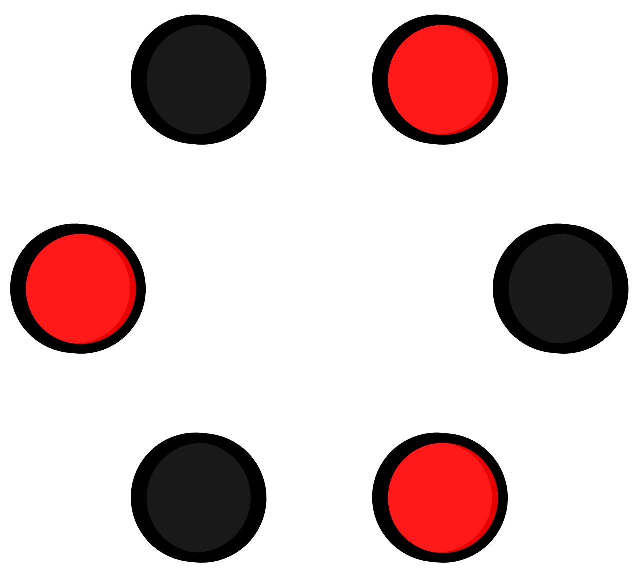
3
3



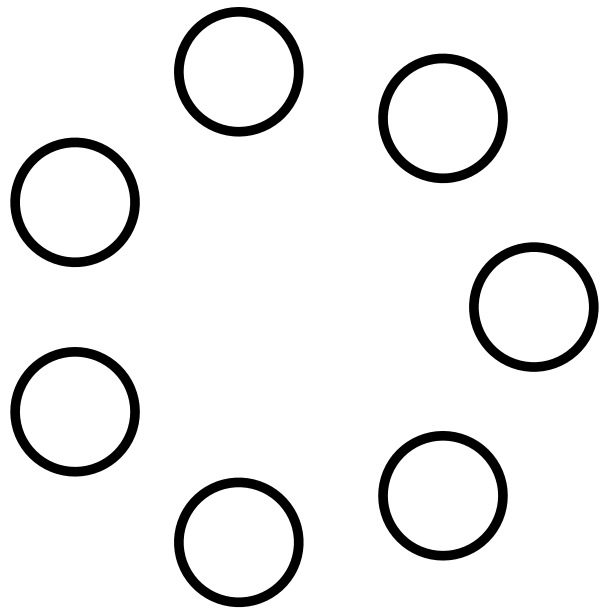
4
3

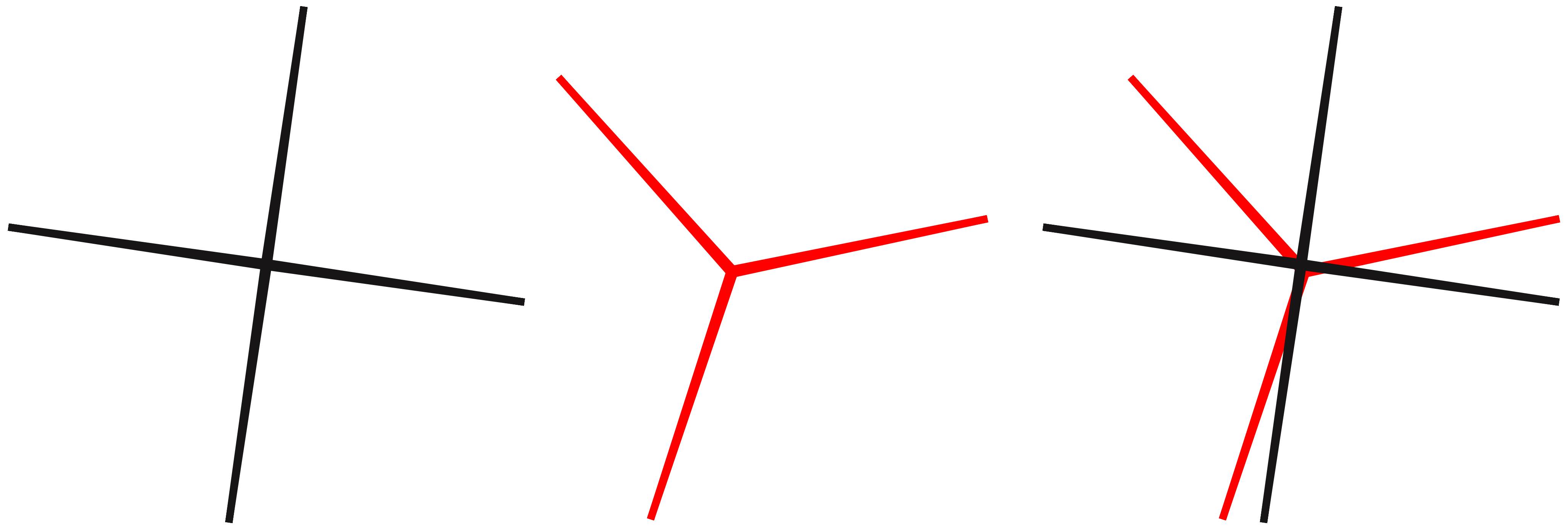


3
3

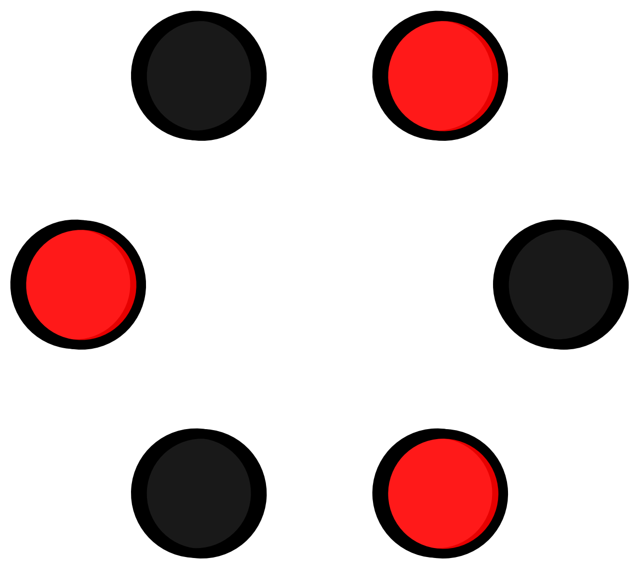


4
3

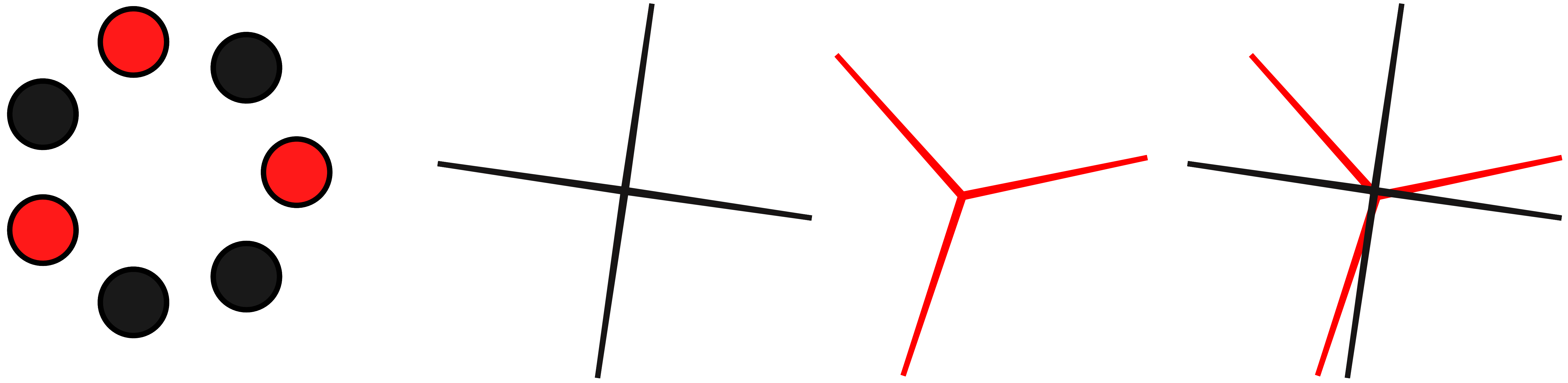




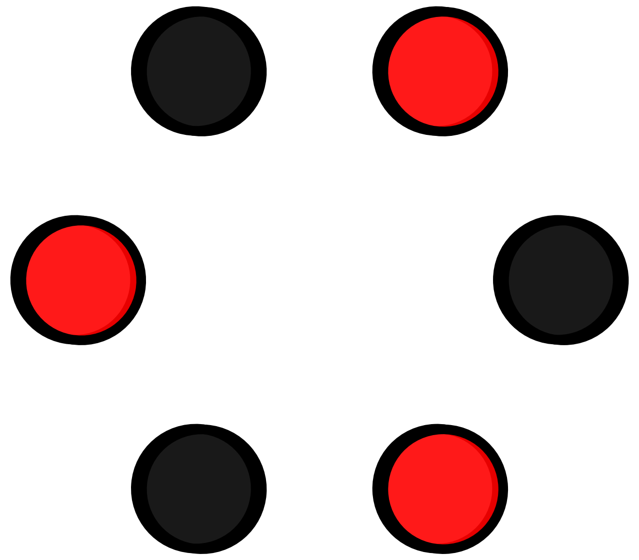
3
3



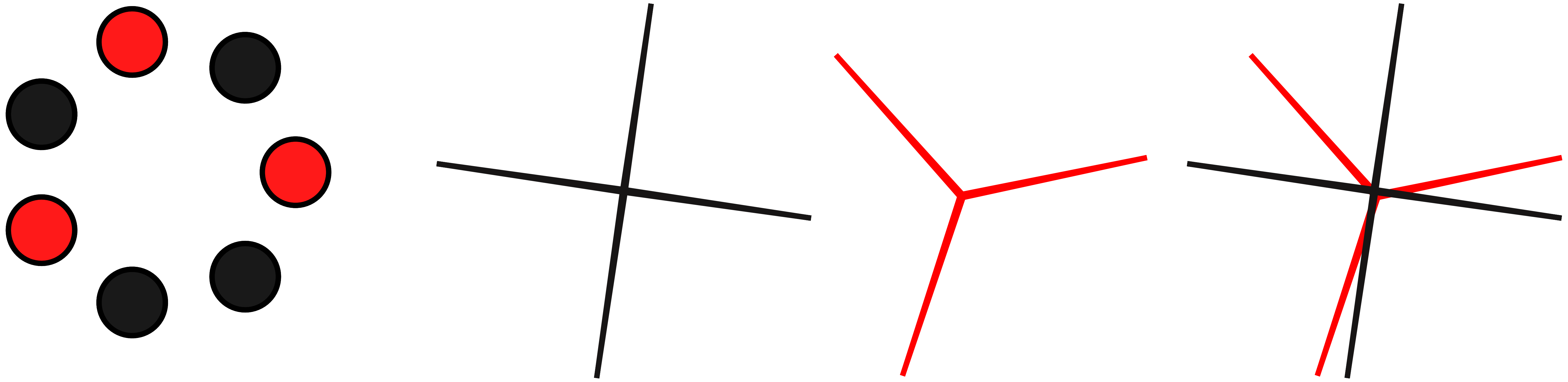
4
3



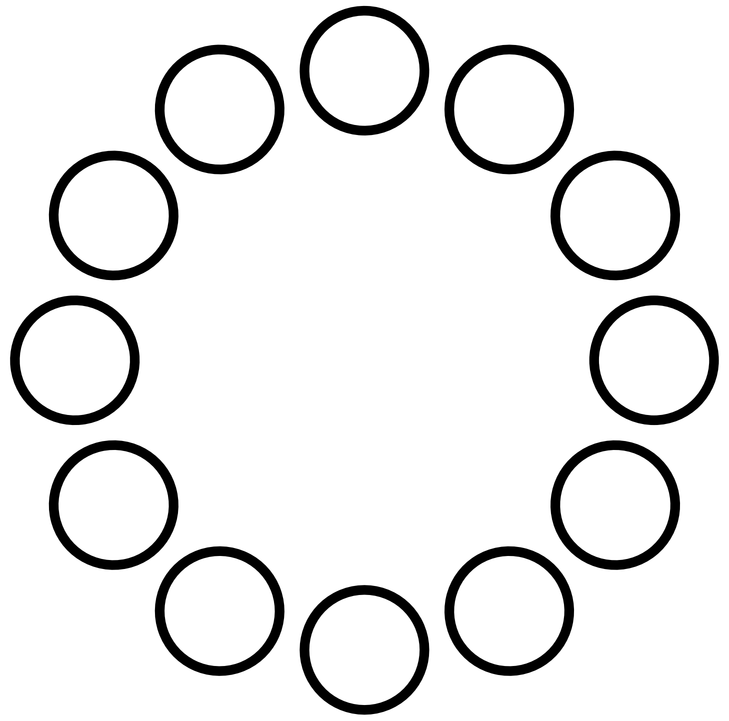
3
3



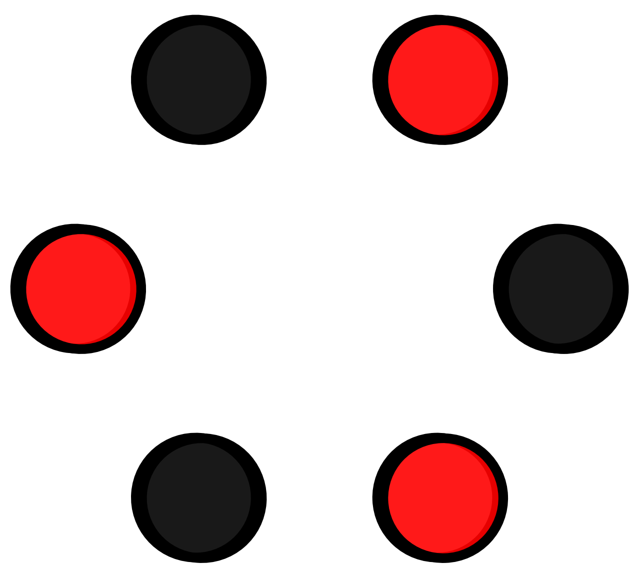
4
3



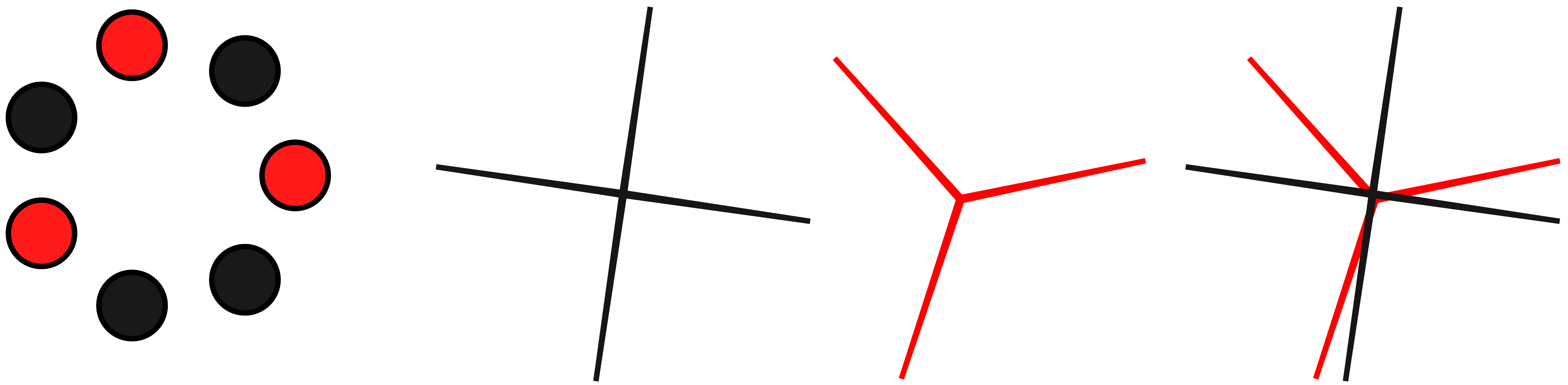
5
7



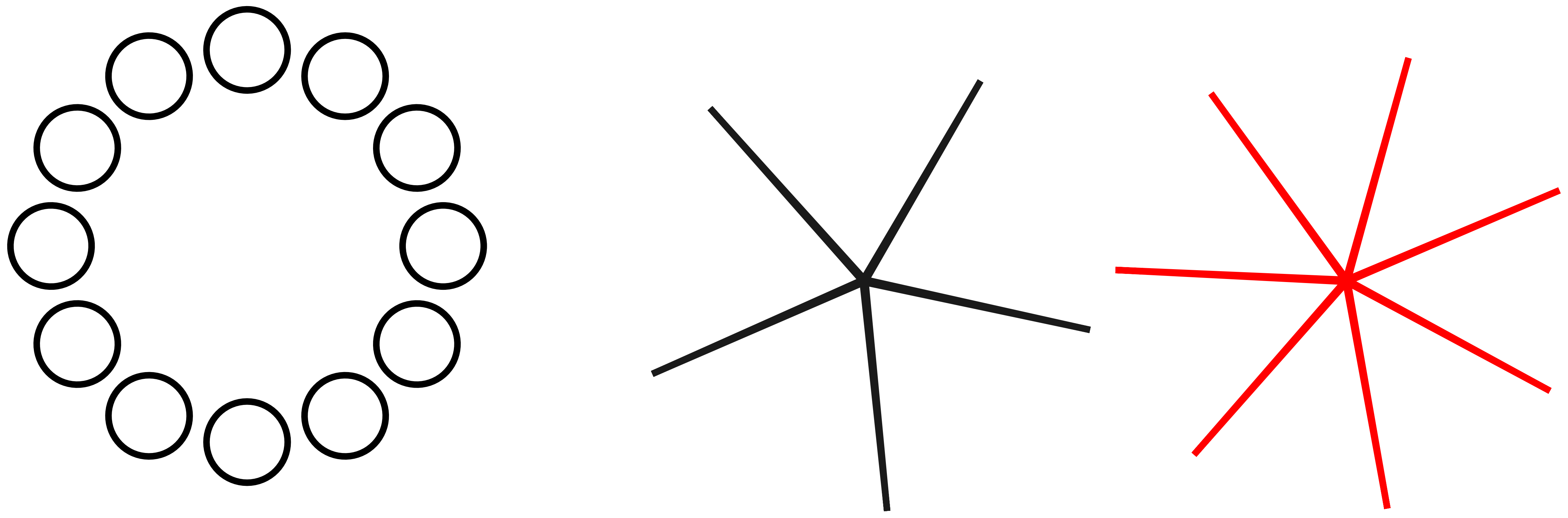
3
3

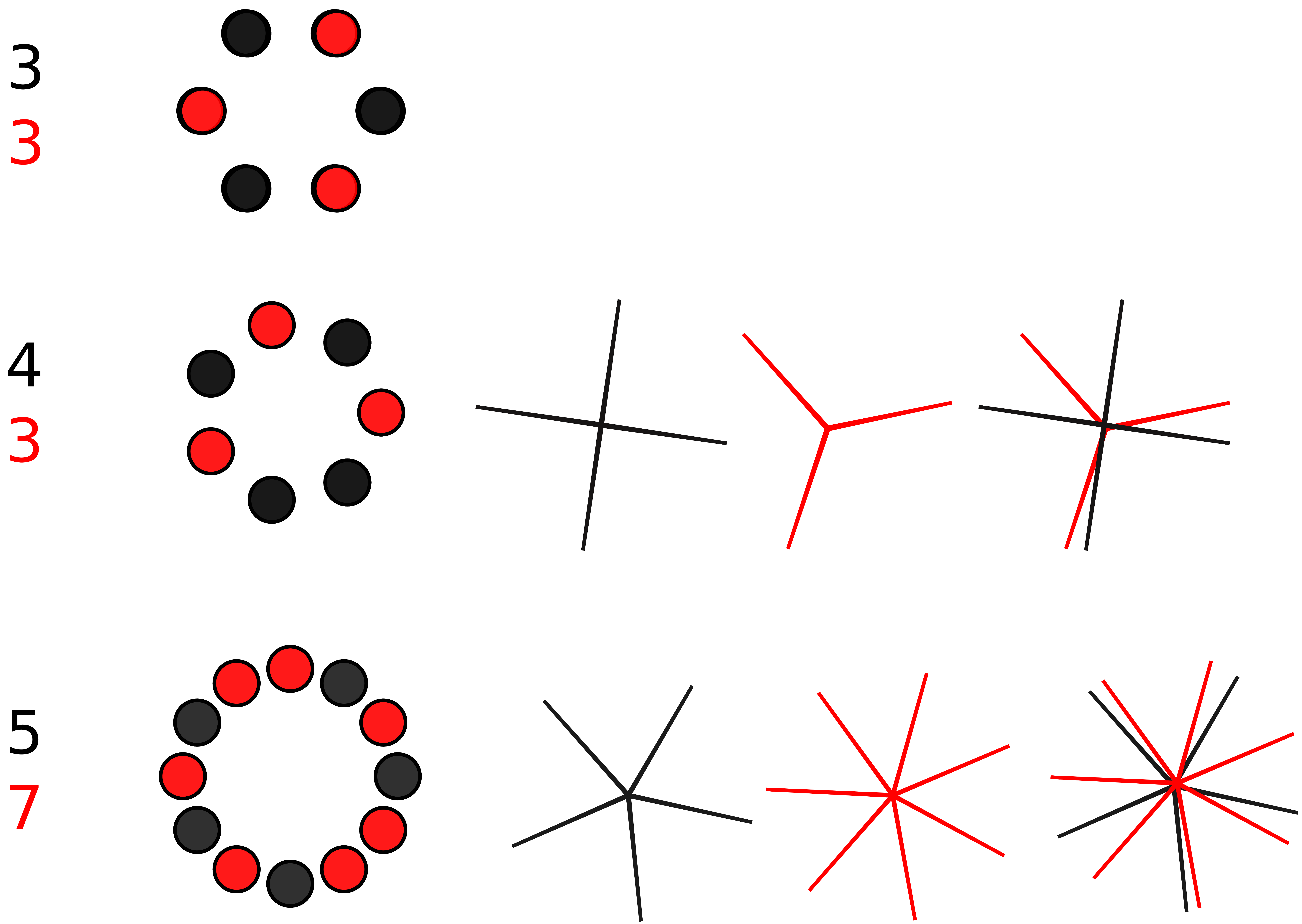


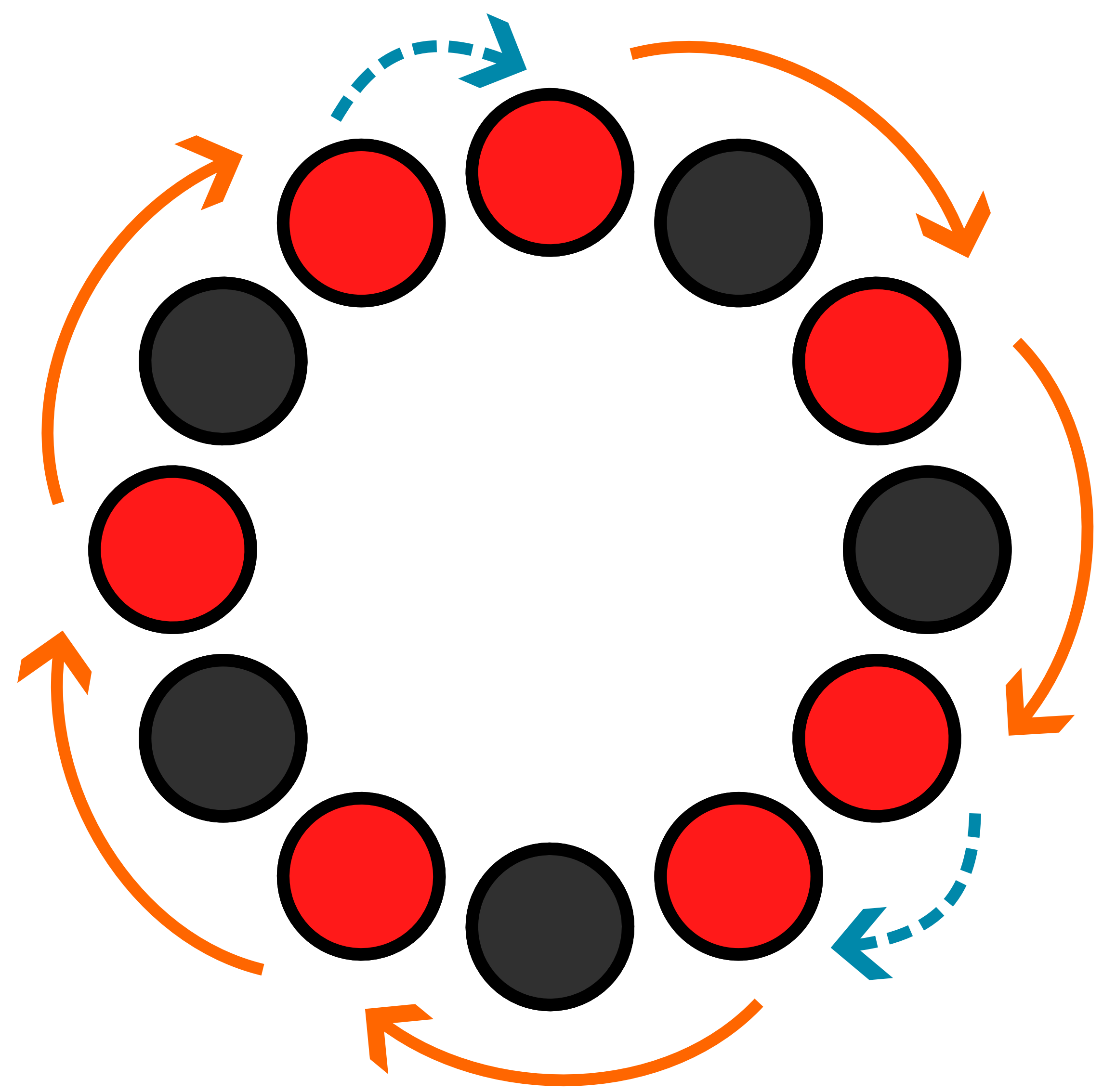
4
3

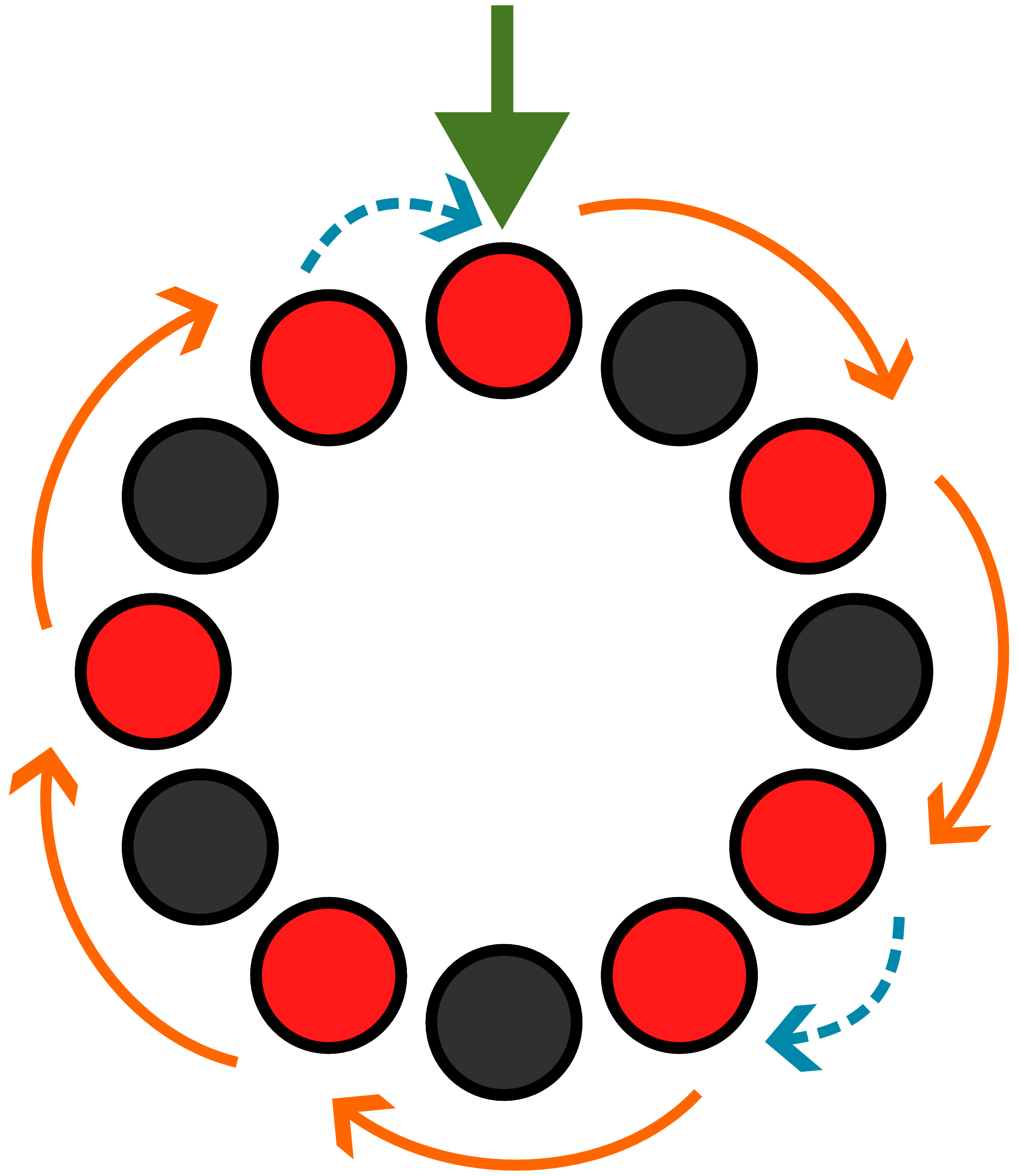


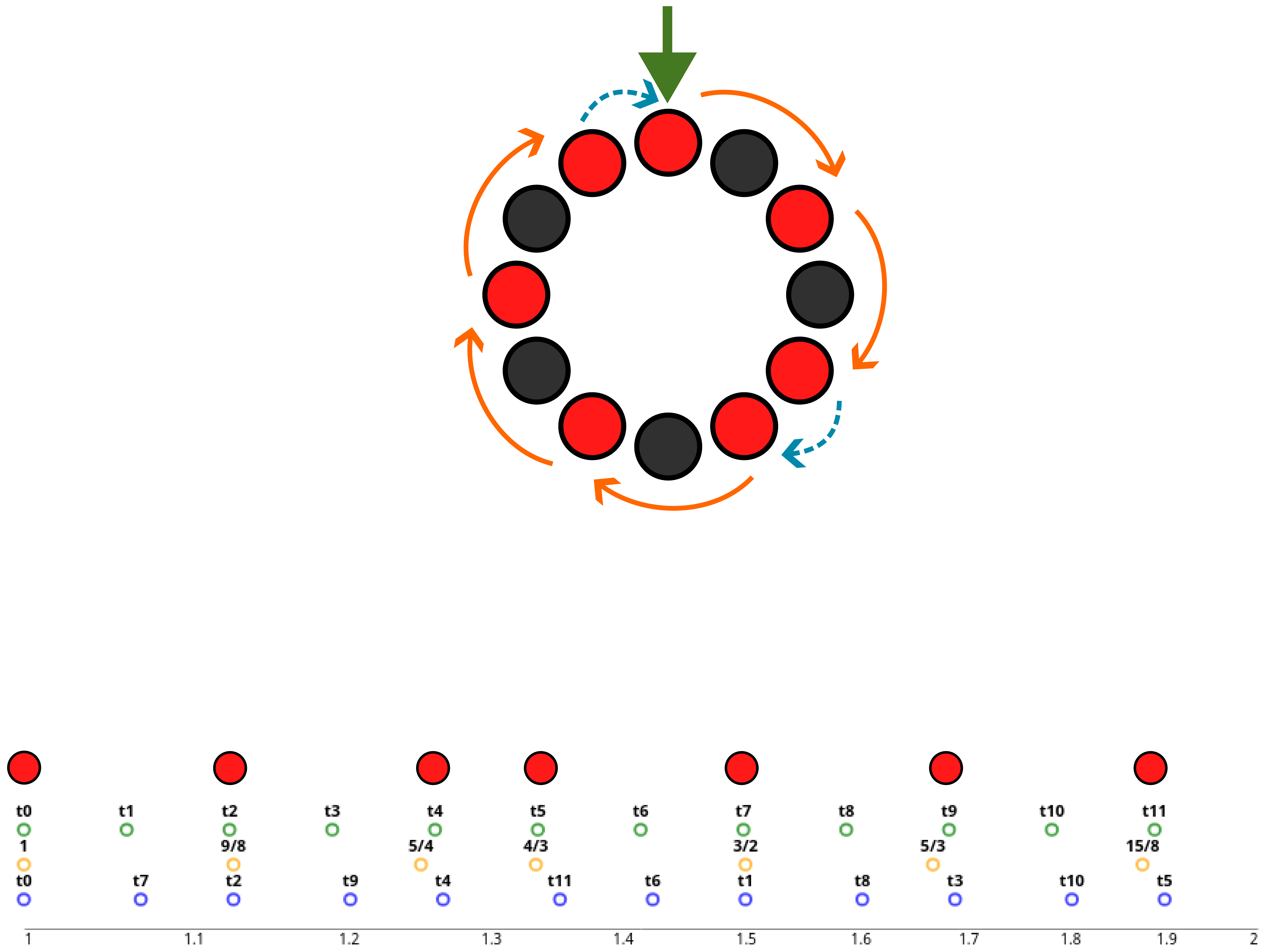
5
7



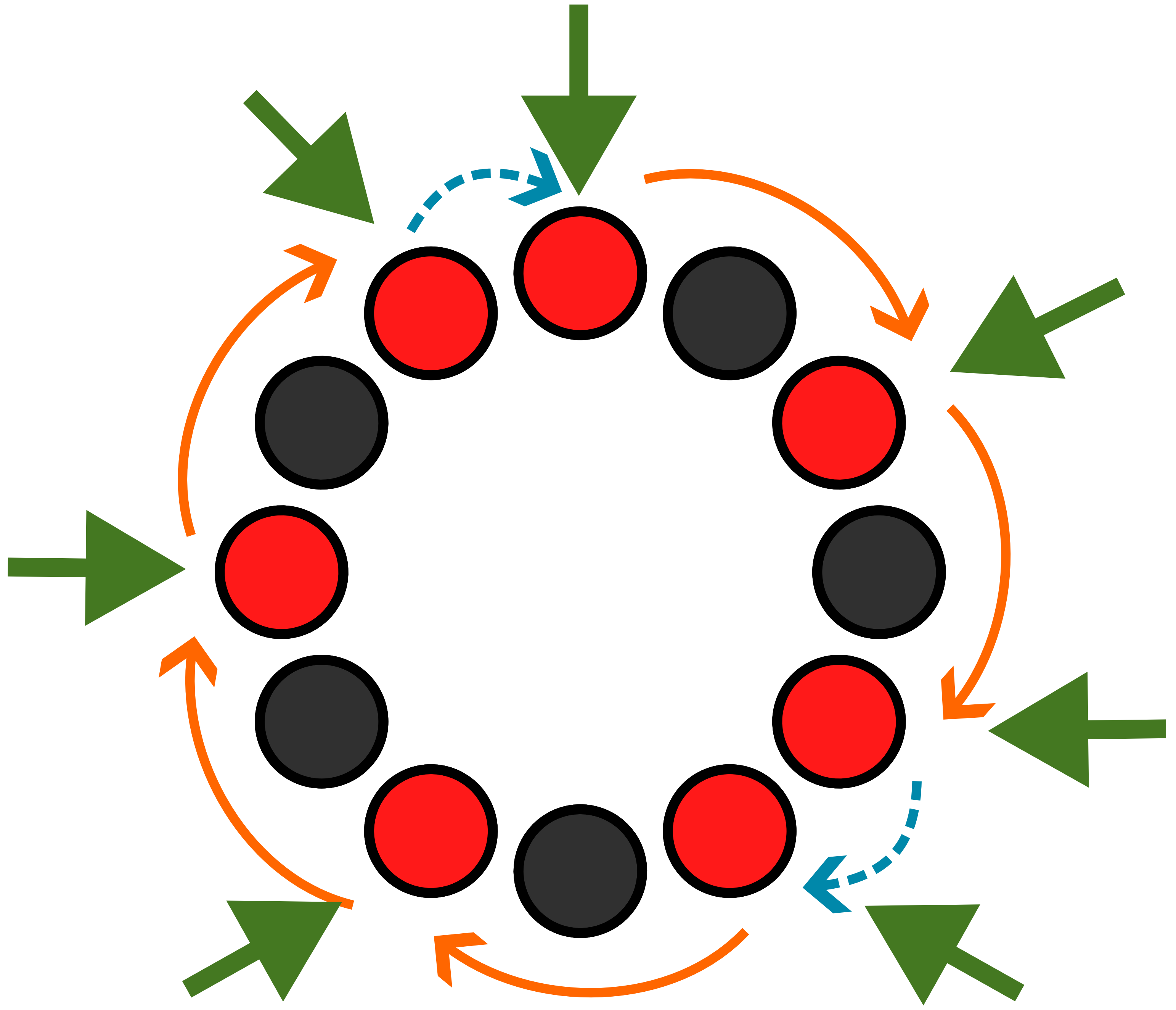

3
3
4
3
5
7

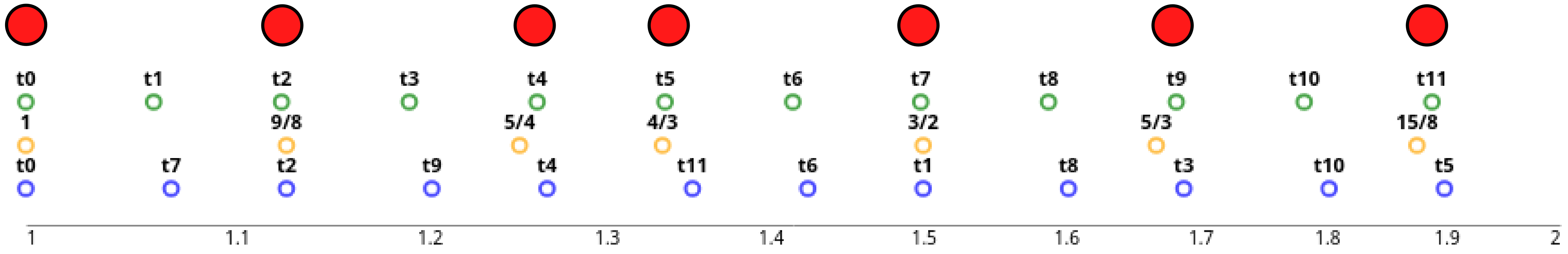


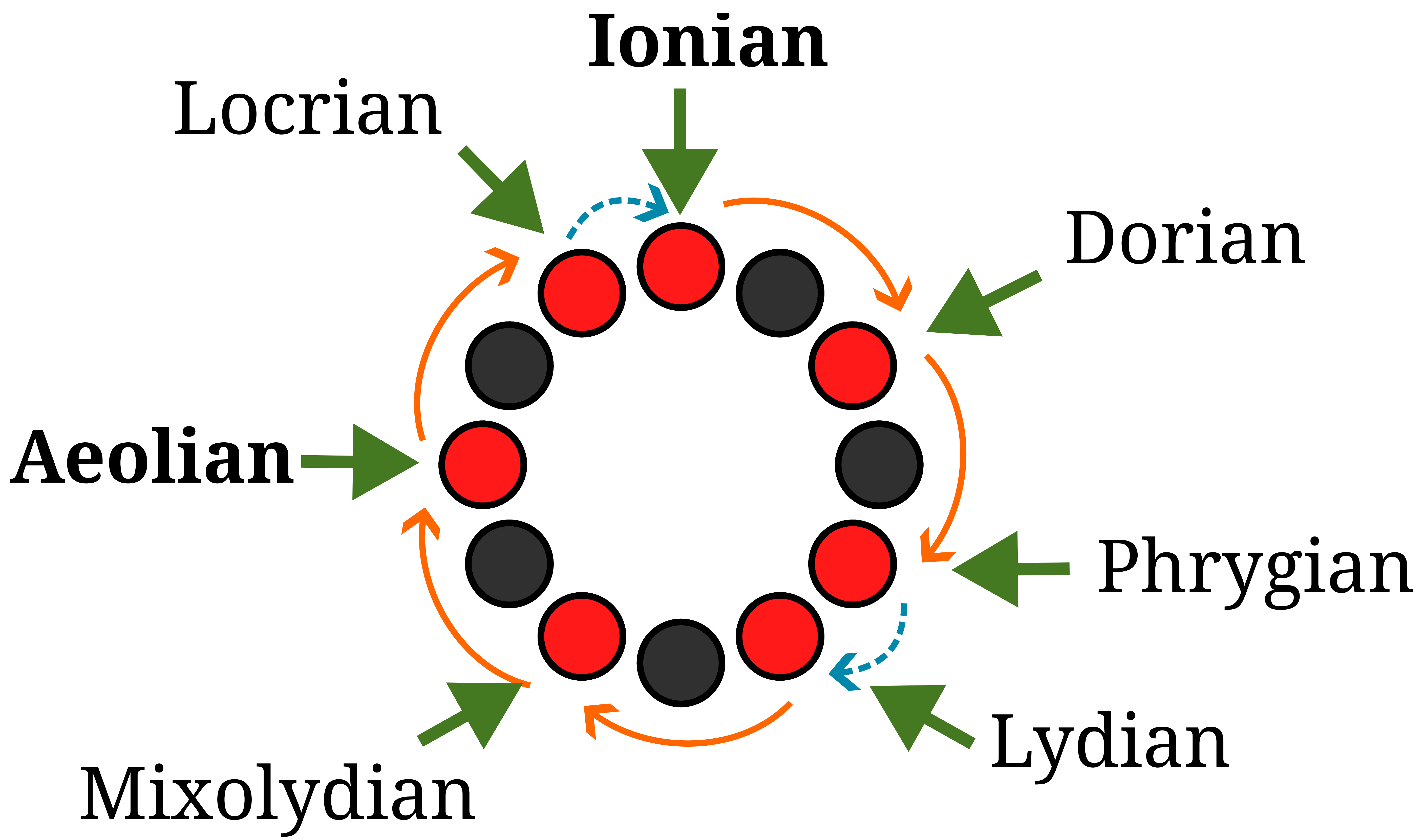


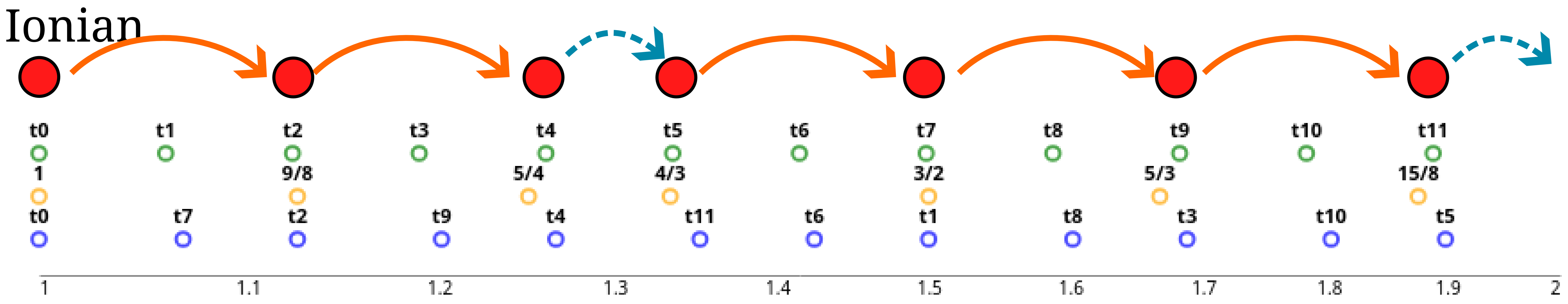


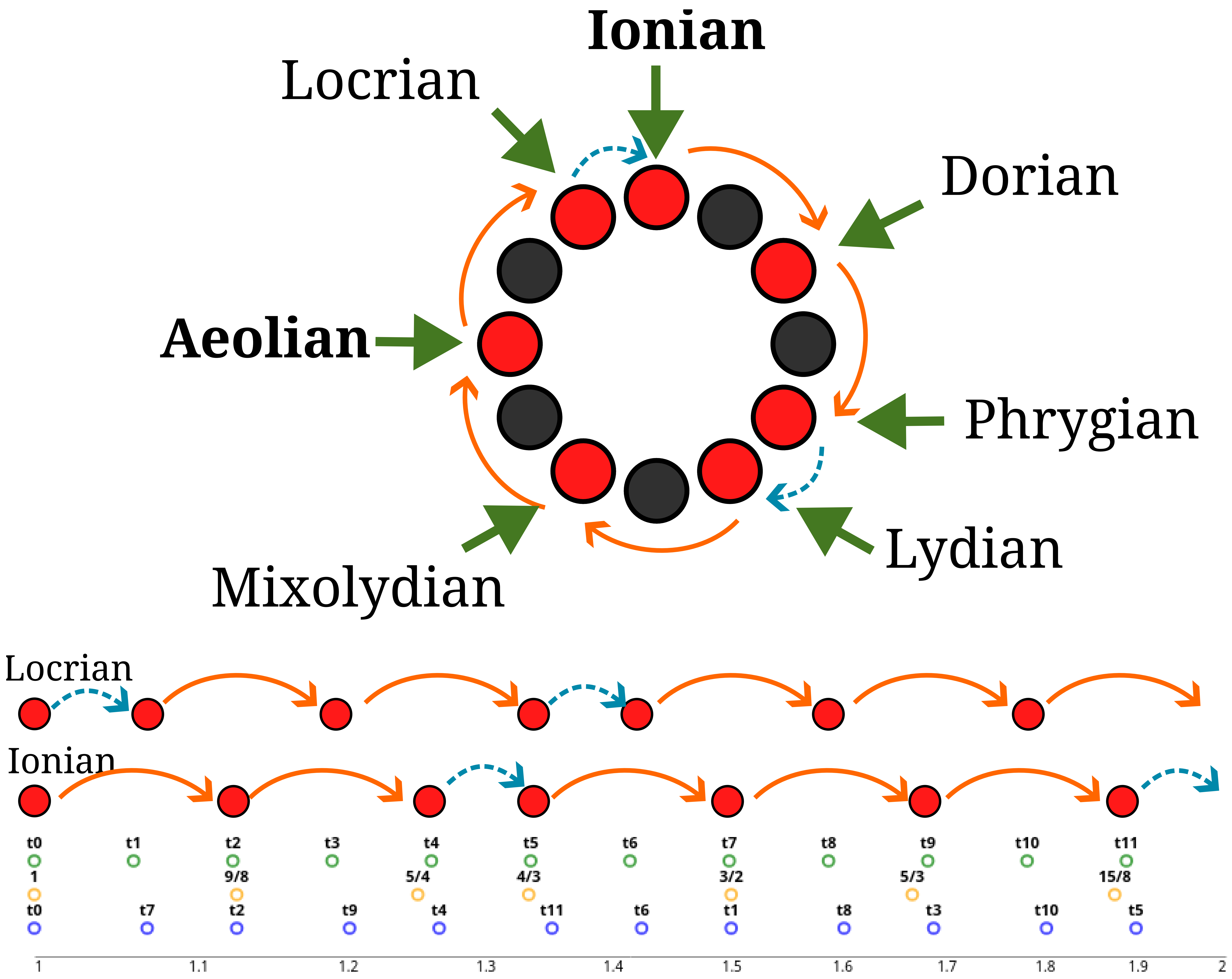
t0
t1
t2
t3
t4
t5
t6
t7
t8
t9
t10
t11
1
9/8
5/4
4/3
3/2
5/3
15/8
t0
t7
t2
t9
t4
t11
t6
t1
t8
t3
t10
t5
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

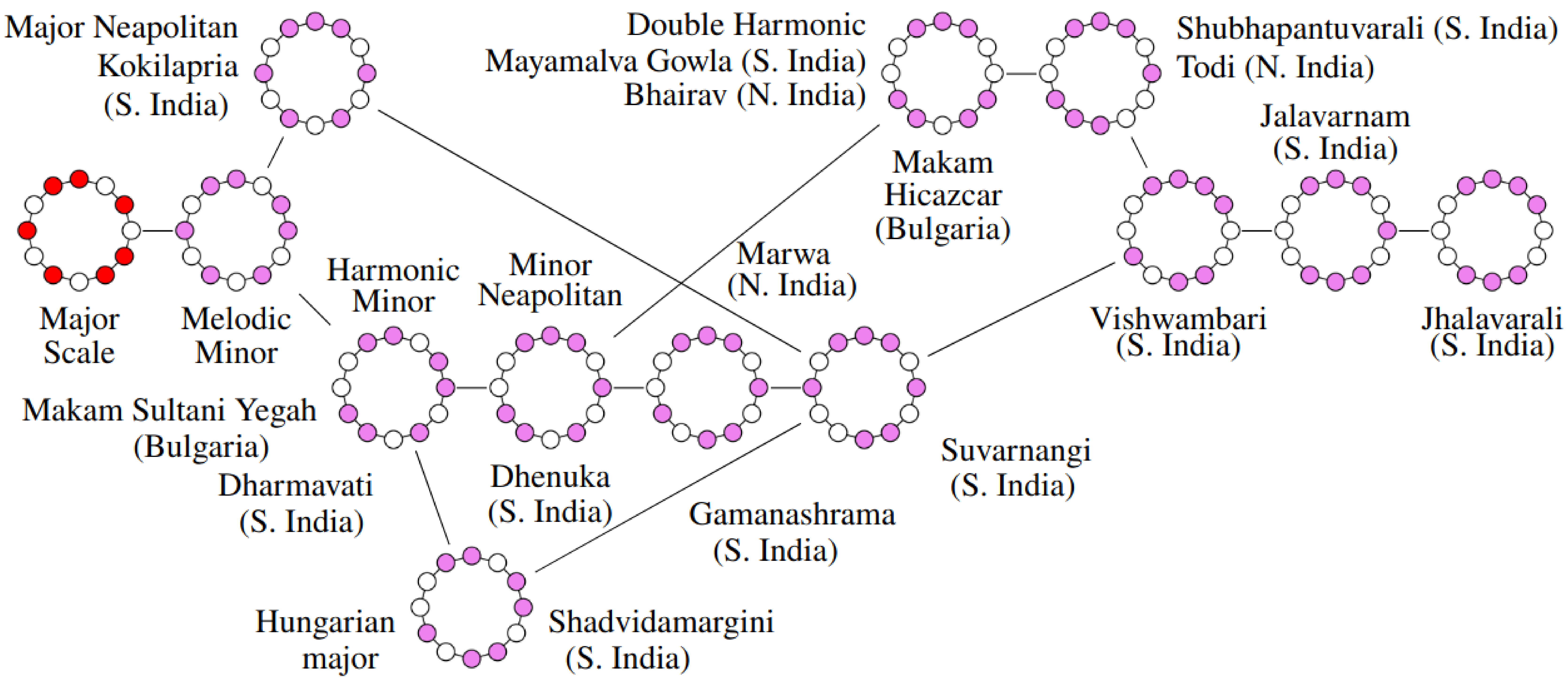



t0 t1 t2 t3 t4 t5 t6 t7 t8 t9 t10 t11
1 9/8 5/4 4/3 3/2 5/3 15/8
t0 t7 t2 t9 t4 t11 t6 t1 t8 t3 t10 t5
1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2


Ionian
Locrian
Dorian
Aeolian
Phrygian
Lydian
Mixolydian


Ionian
t0 t1 t2 t3 t4 t5 t6 t7 t8 t9 t10 t11
1 9/8 5/4 4/3 3/2 5/3 15/8
t0 t7 t2 t9 t4 t11 t6 t1 t8 t3 t10 t5
1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2

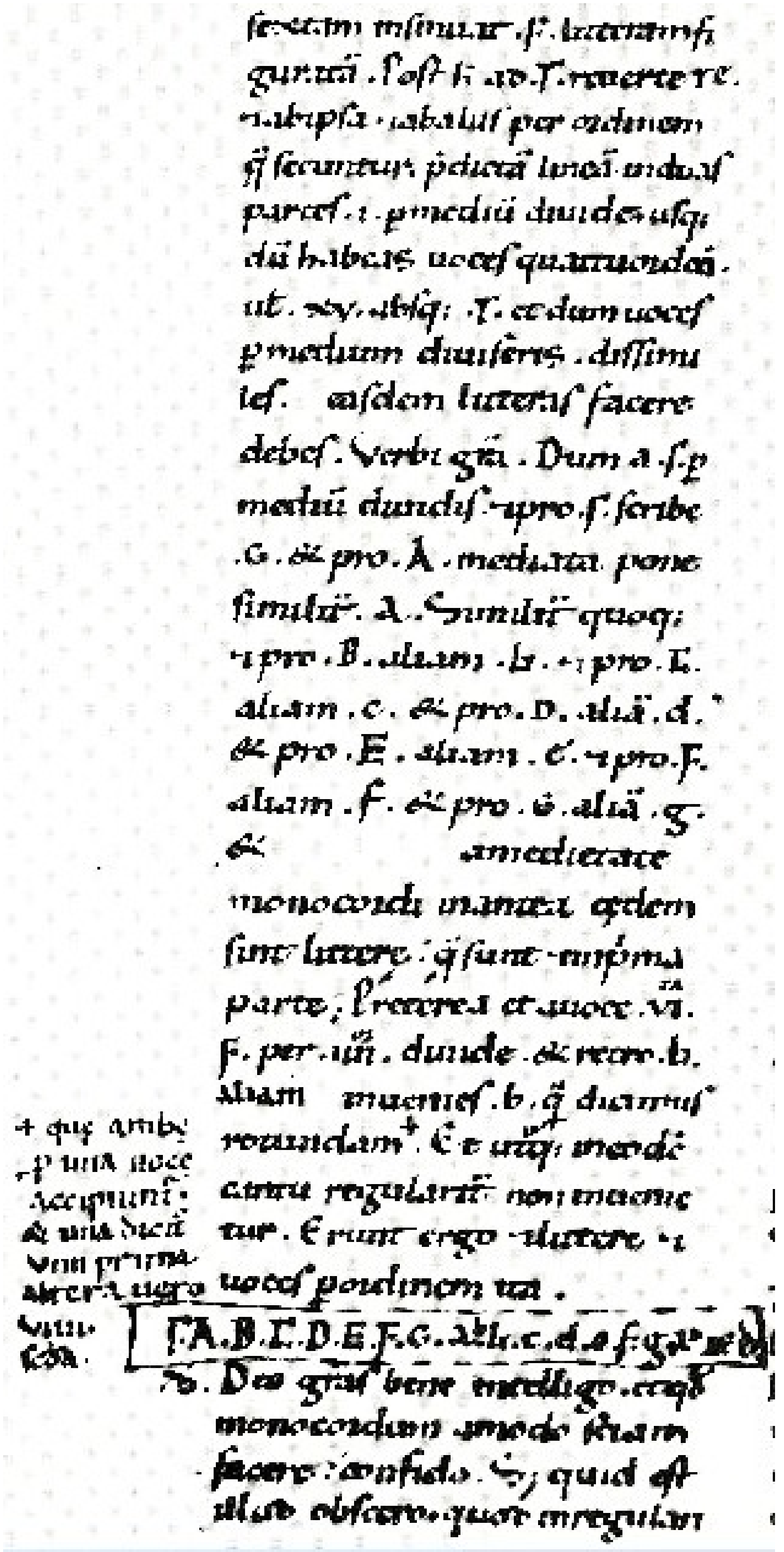

Ionian
Locrian
Dorian
Aeolian
Phrygian
Lydian
Mixolydian
Locrian
Ionian
t0 t1 t2 t3 t4 t5 t6 t7 t8 t9 t10 t11
1 9/8 5/4 4/3 3/2 5/3 15/8
t0 t7 t2 t9 t4 t11 t6 t1 t8 t3 t10 t5
1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2


Major Neapolitan
Kokilapria
(S. India)
Double Harmonic
Mayamalva Gowla (S. India)
Bhairav (N. India)
Shubhapantuvarali (S. India)
Todi (N. India)
Jalavarnam
(S. India)
Makam
Hicazcar
(Bulgaria)
Major
Scale
Melodic
Minor
Harmonic
Minor
Minor
Neapolitan
Marwa
(N. India)
Vishwambari
(S. India)
Jhalavarali
(S. India)
Makam Sultani Yegah
(Bulgaria)
Dharmavati
(S. India)
Dhenuka
(S. India)
Gamanashrama
(S. India)
Suvarnangi
(S. India)
Hungarian
major
Shadvidamargini
(S. India)

A, B, C, D, E, F, G

Pseudo Odo

Pseudo Odo



sextam insinuat .f. litteram fi-
guratā. Post h' ad .Γ. reuerte re-
ate ipsa tabulas per ordinem
q̄ secuntur, p̄dictā lineā in duas
partes .i. p medium diuides usq;
dū habeas uoces quatuordecī.
ut . xiv . absq; .Γ. et dum uoces
p medium diuiseris . dissimi
les . easdem litteras facere
debes. Verbi gr̄a . Dum a .Γ. p
mediū diuidis . pro .Γ. scribe
.G. et pro .A. mediatā pone
similit'. a. Similit' quoq;
pro .B. aliam .b. pro .C.
aliam .c. et pro .D. aliā .d.
et pro .E. aliam .e. pro .F.
aliam .f. et pro .G. aliā .g.
et [illegible] a medietate
monocordi inantea eędem
sunt litterę ; q̄ sunt in prima
parte ; Preterea et a uoce .vi.
F. per .iiii. diuide . et recte .b.
aliam inuenies .b. q̄ dicimus
rotundam. Et e utraq; in eodē
cantu regulariter non inueni-
tur . Erunt ergo litterę et
uoces p ordinem ita .

+ que ambe p una uoce accipiunt . et una dicitur . Vna prima altera [illegible] [illegible]

Γ.A.B.C.D.E.F.G.a.b.c.d.e.f.g.a [illegible]

D. Deo gratias bene intelligo . ergo
monocordum amodo etiam
facere confido. M. quid est
illud obsecro . quod inregulari

A4 = 440Hz
ISO 16 (1975)

A, B, C, D, E, F, G

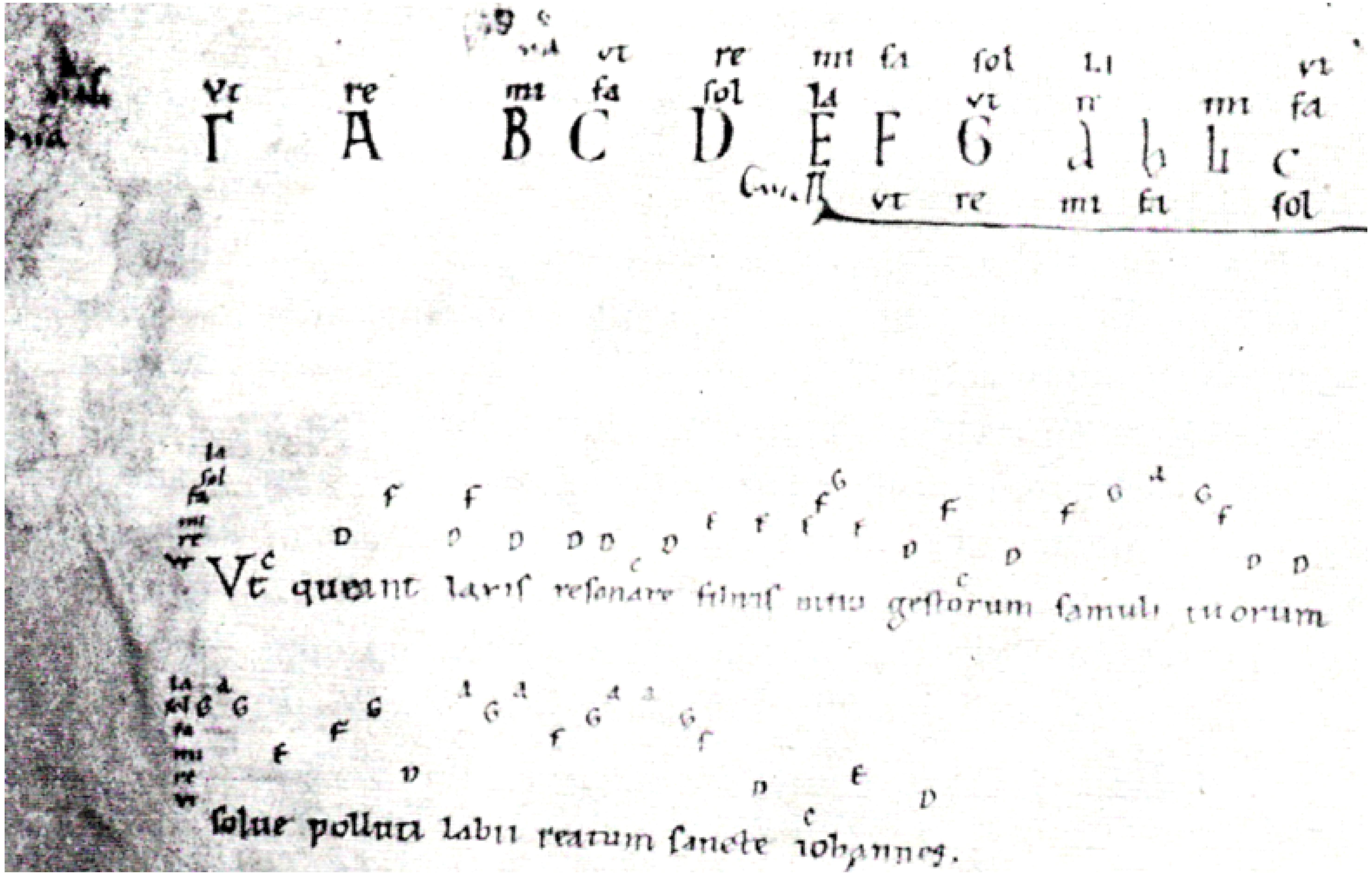
C, D, E, F, G, A, B

C, D, E, F, G, A, **H**

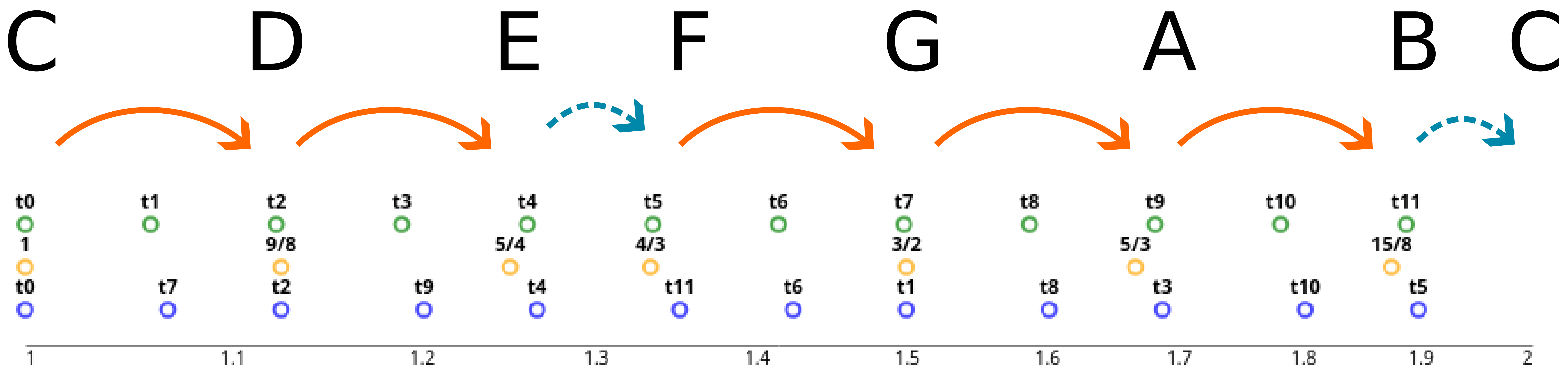
C, D, E, F, G, A, **H**

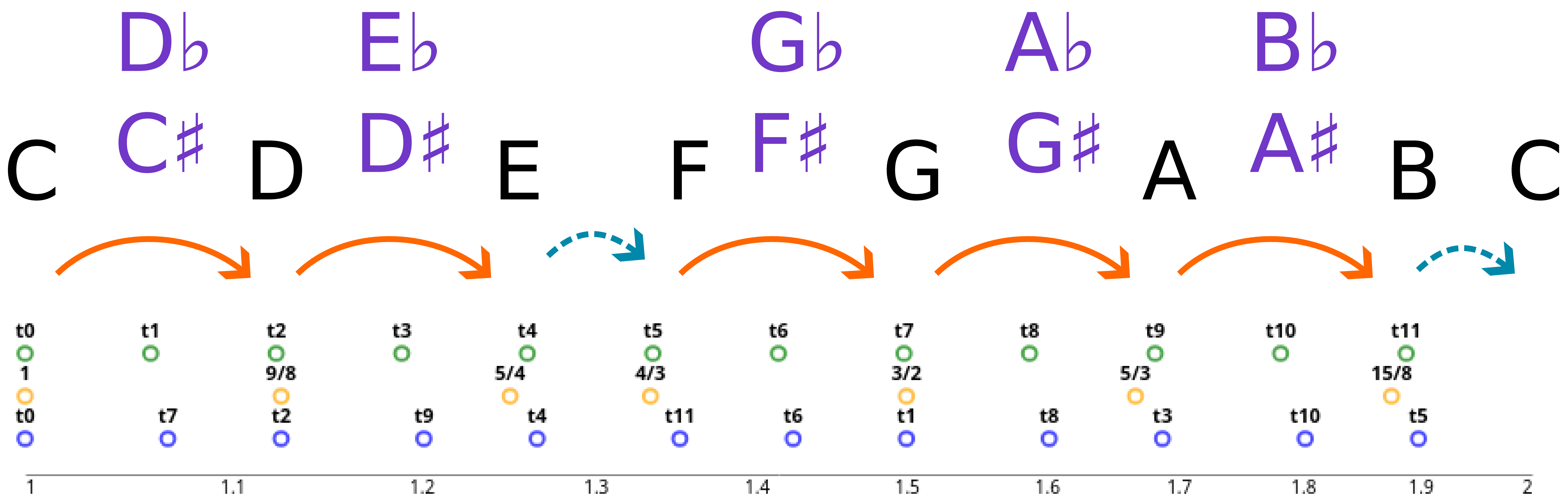
𝔟 𝔥

C, D, E, F, G, A, H

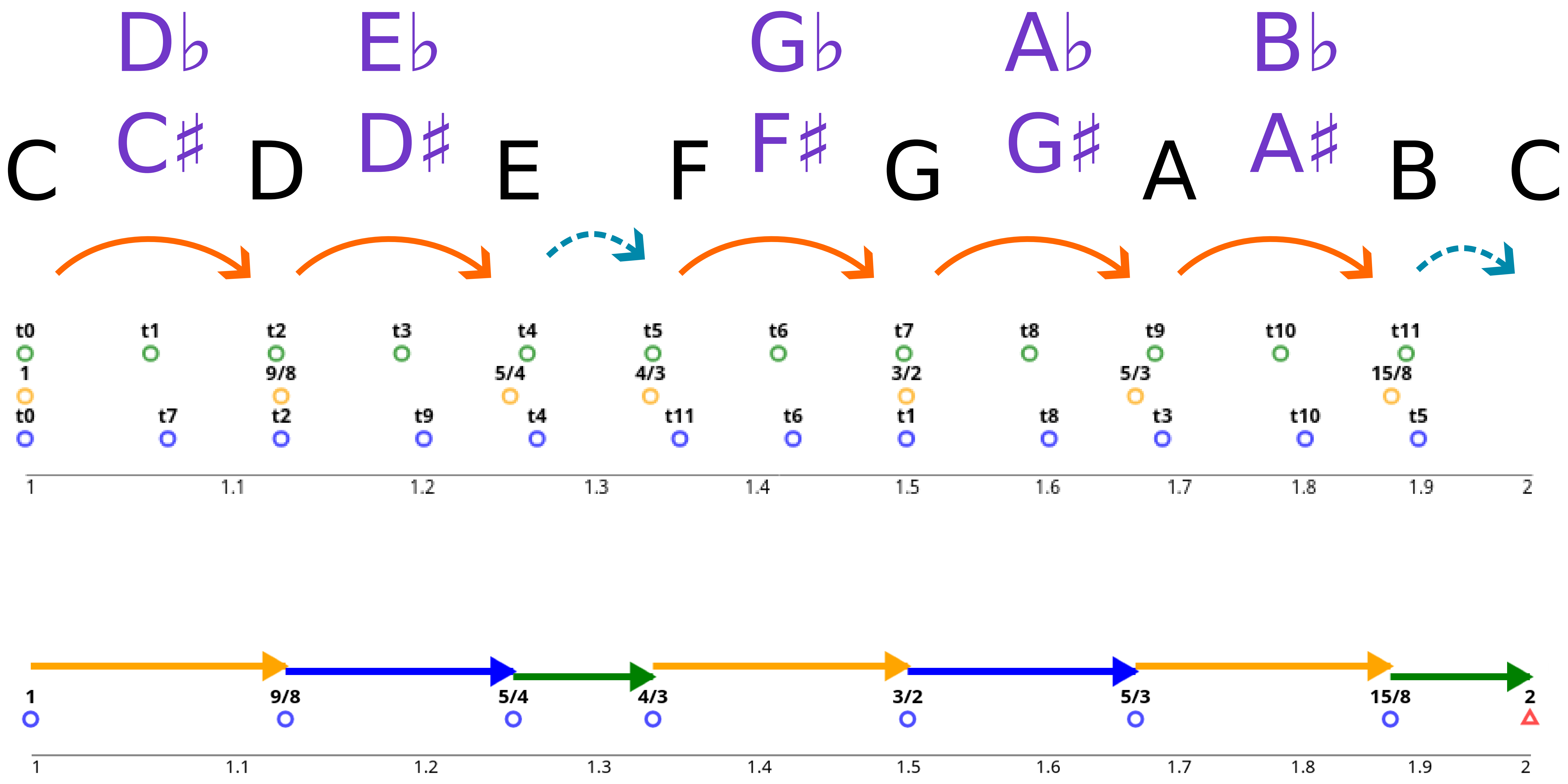

Vt queant laxis resonare fibris mira gestorum famuli tuorum
solue polluta labii reatum sancte iohannes.

C, D, E, F, G, A, **B**


C
D
E
F
G
A
B
C
t0
t1
t2
t3
t4
t5
t6
t7
t8
t9
t10
t11
1
9/8
5/4
4/3
3/2
5/3
15/8
t0
t7
t2
t9
t4
t11
t6
t1
t8
t3
t10
t5
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

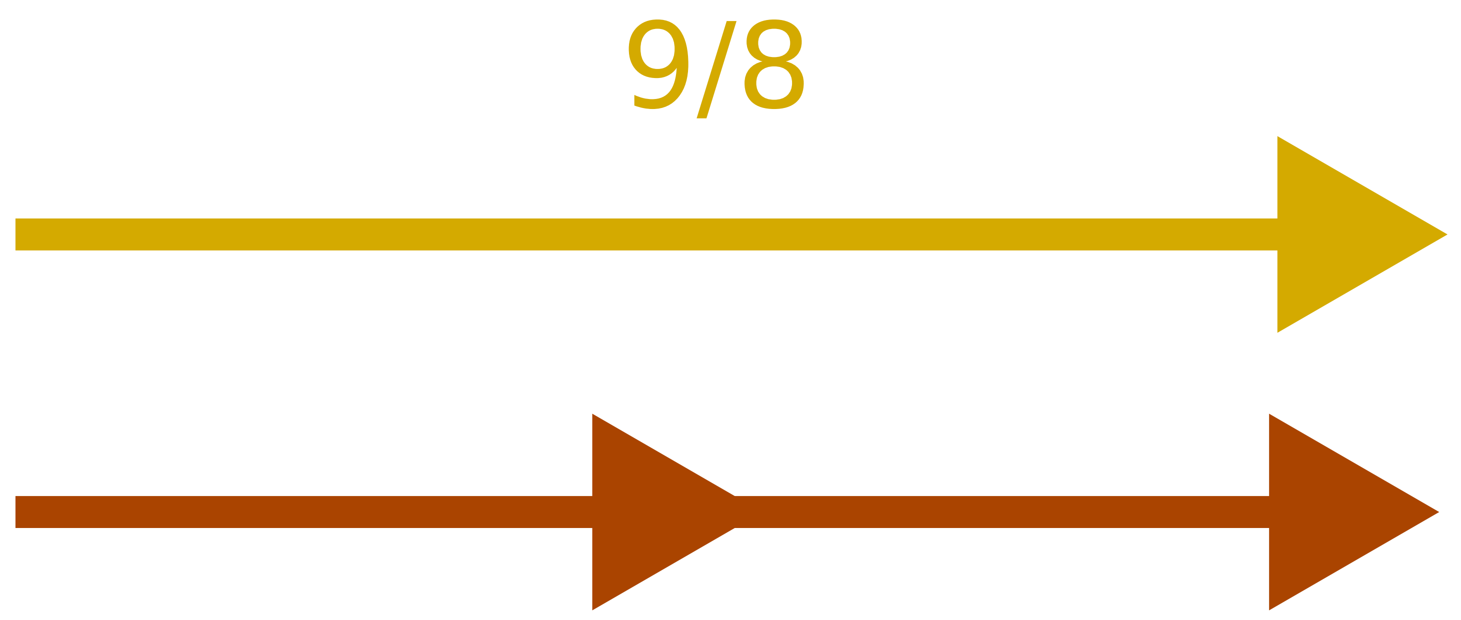


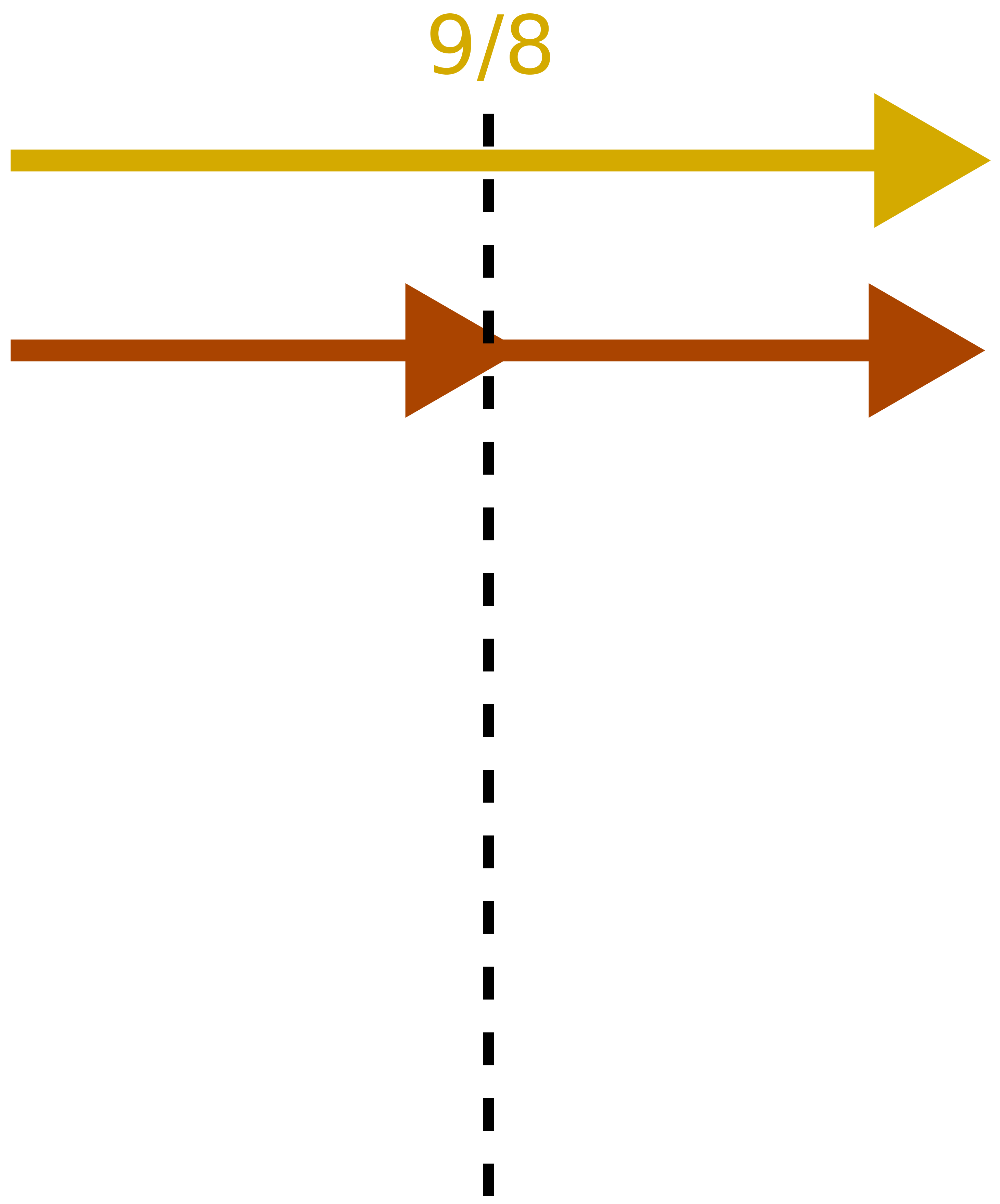
D♭ E♭ G♭ A♭ B♭
C♯ D♯ F♯ G♯ A♯
C D E F G A B C
t0 t1 t2 t3 t4 t5 t6 t7 t8 t9 t10 t11
1 9/8 5/4 4/3 3/2 5/3 15/8
t0 t7 t2 t9 t4 t11 t6 t1 t8 t3 t10 t5
1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2

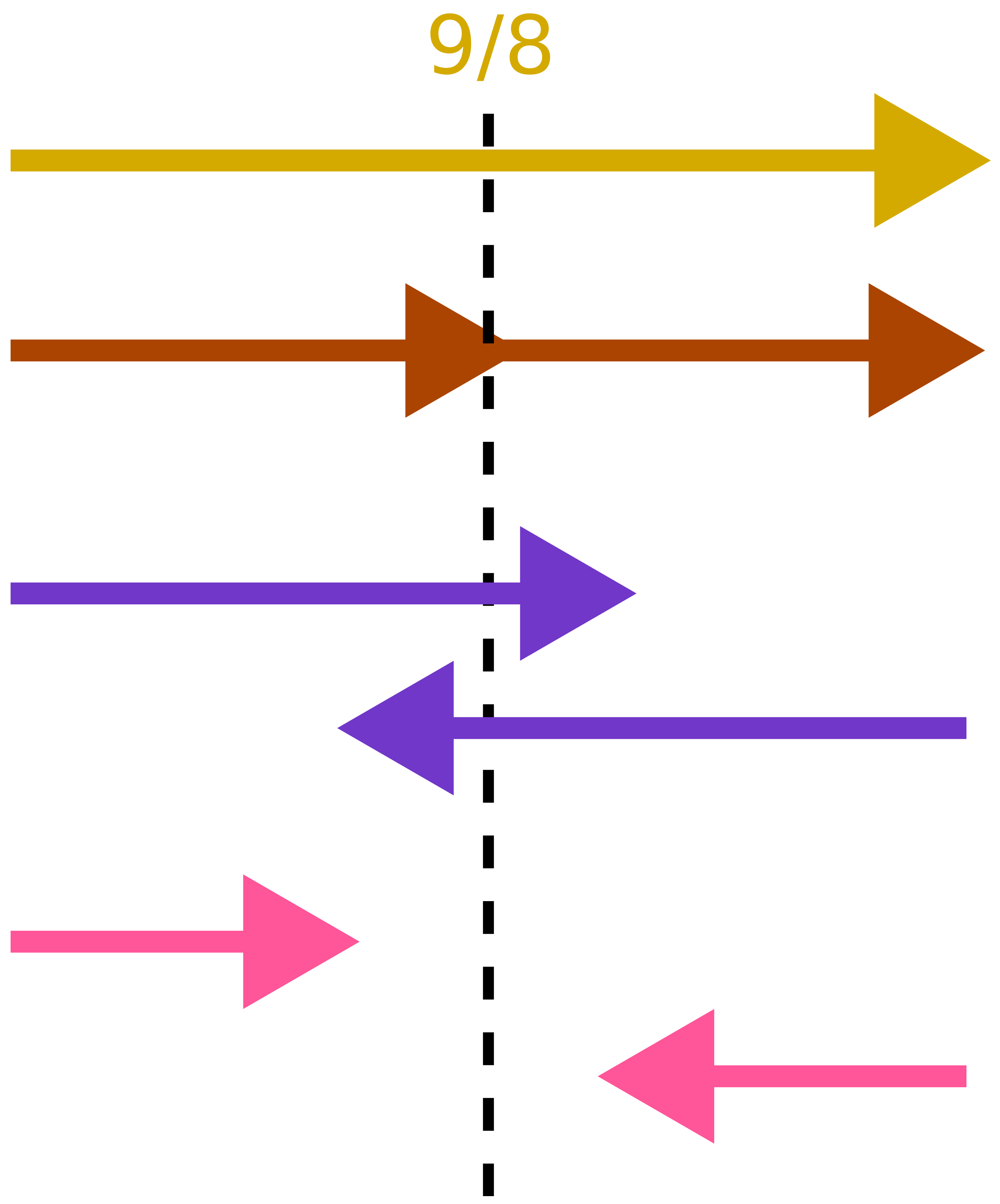


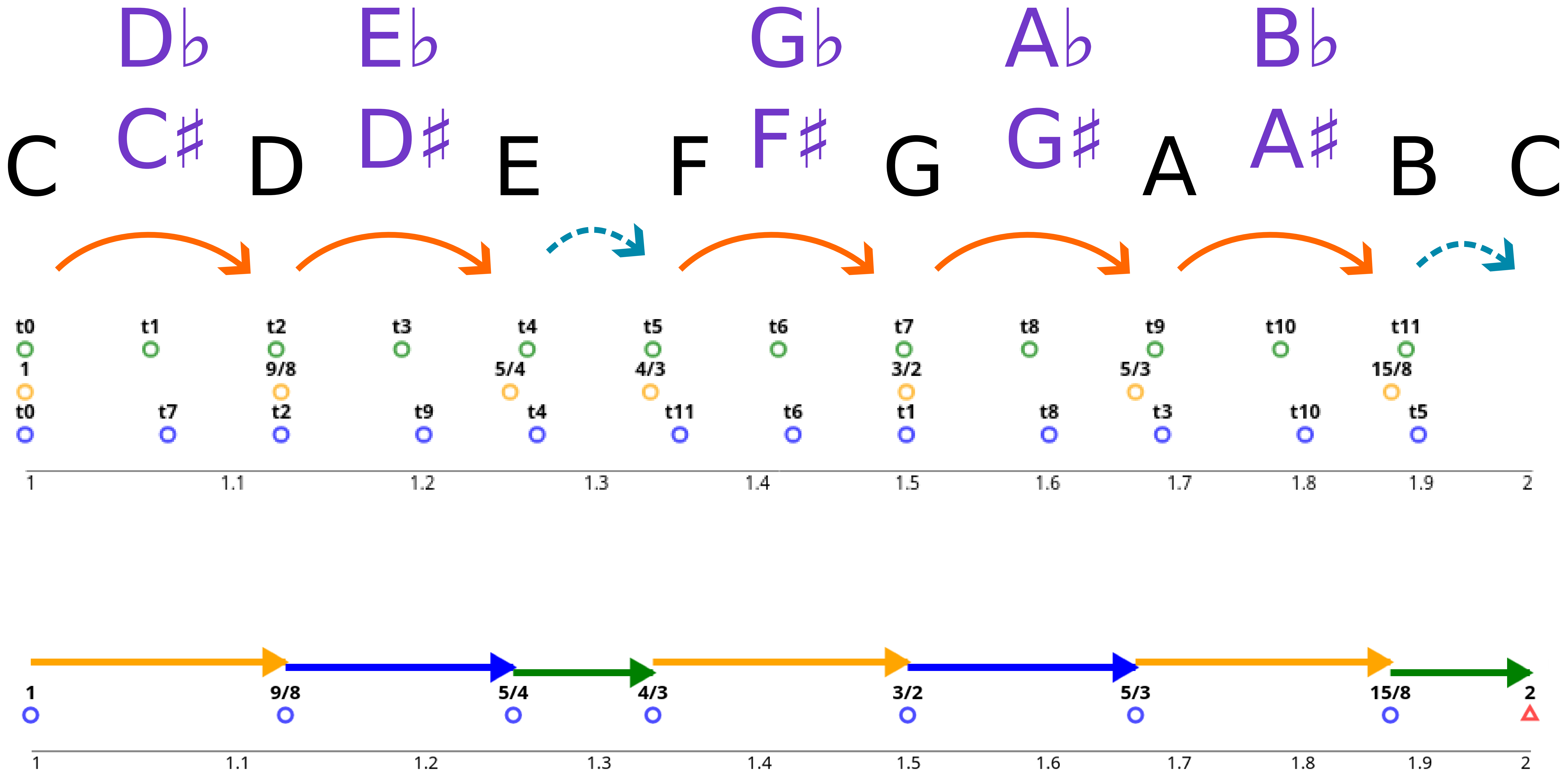
$$\frac{9}{8} \qquad \frac{10}{9} \qquad \frac{16}{15}$$


9/8

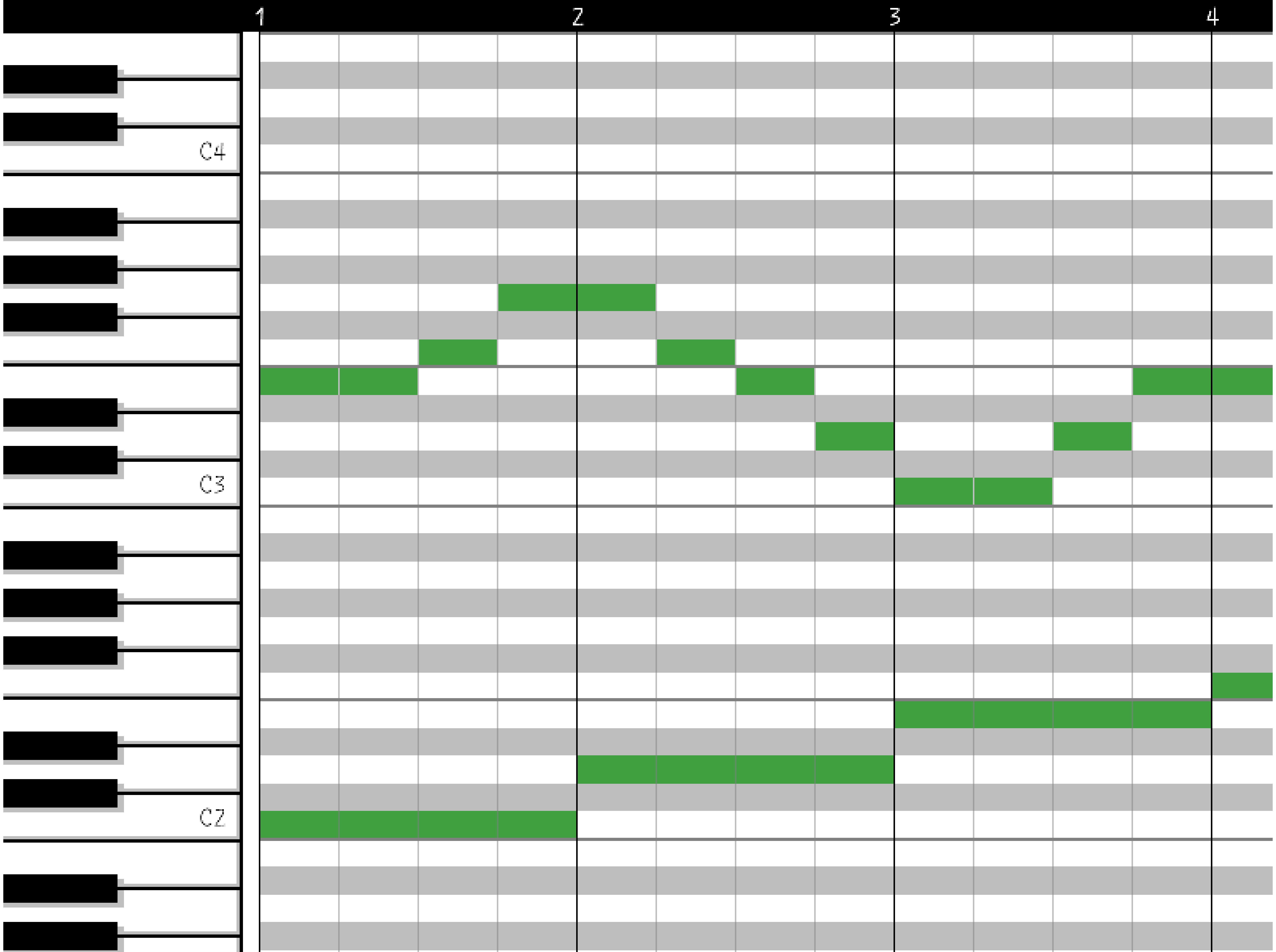

9/8


9/8


9/8

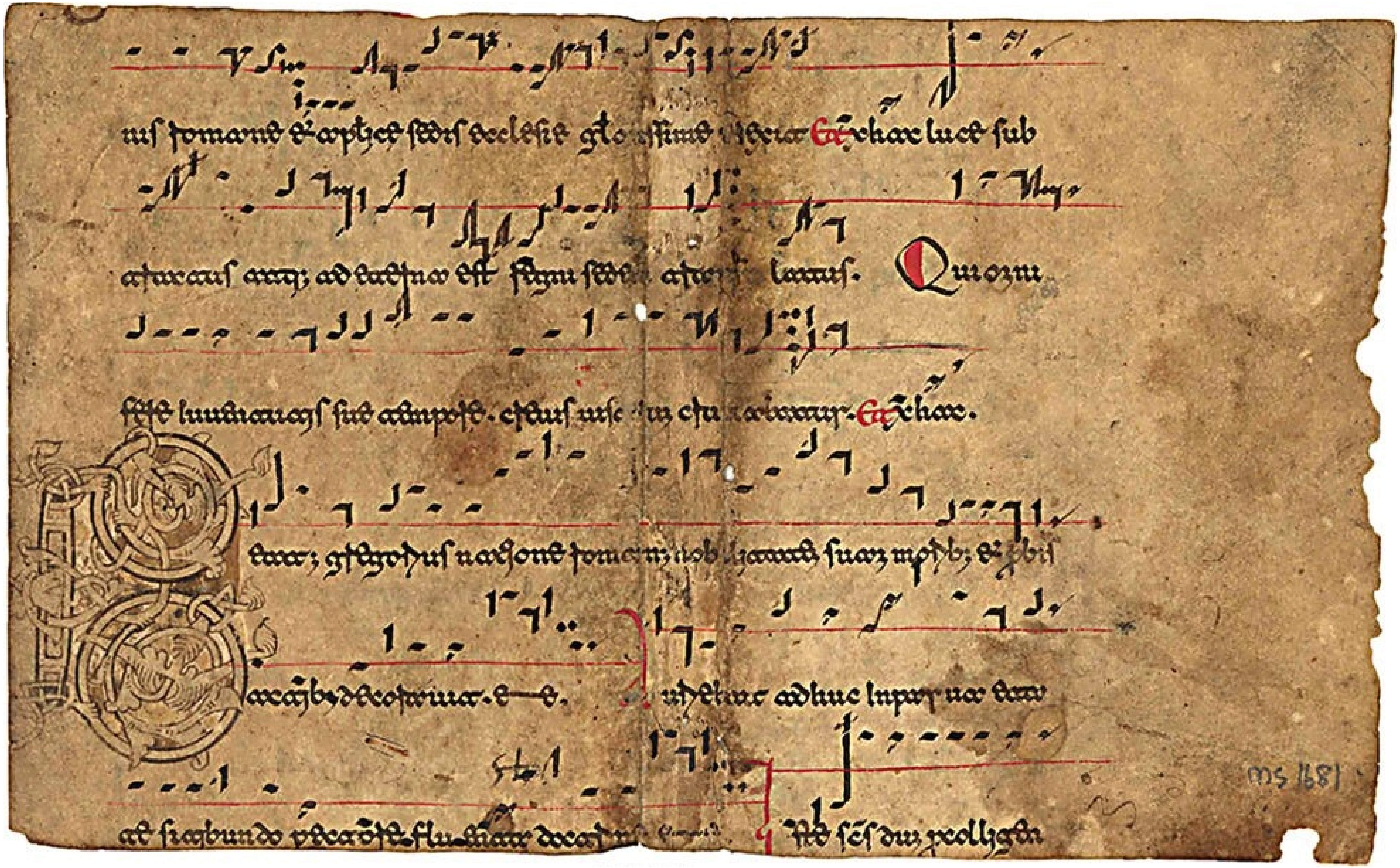


$\frac{9}{8}$ $\frac{10}{9}$ $\frac{16}{15}$


1
2
3
4
C4
C3
C2

d dfgh g g g gf gff ded ffg
me sal uum fac ser uum tu um de

hg hgfg gf f ghg h ge g eg f deded
tem in te mise rere mihi domi



hac luce sub
Quorum
Et hac.
adhuc
ms 1681

AD MISSAM. INTROITUS
S I-TI-VIT
a-nima me-a
ad De-um fontem
vivum quando ve-
ni-am et appa-re-
bo ante fa-ci-em
Dei. Que-mad-
modum desiderat

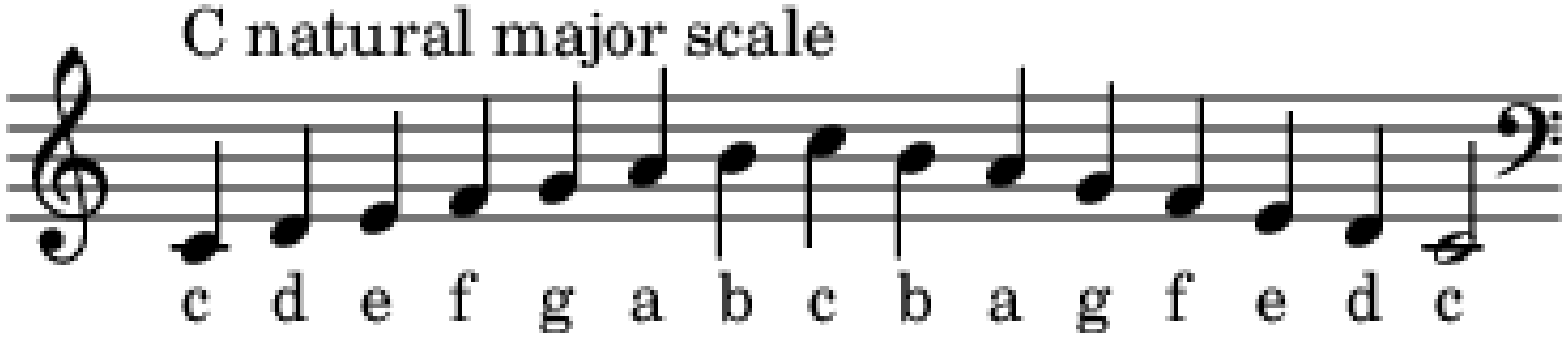
PRELUDE

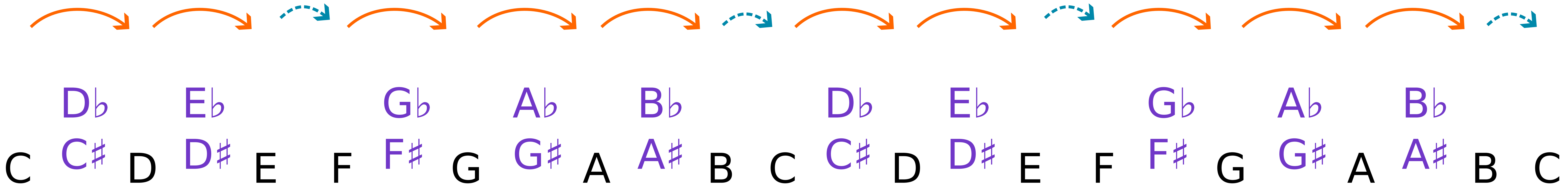
Op. 28, No. 7

Frederic Chopin

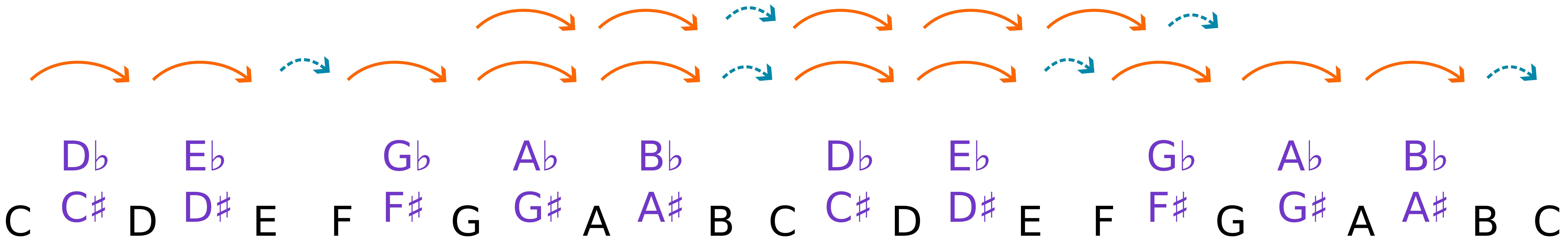
Techno

Drumline Cadence

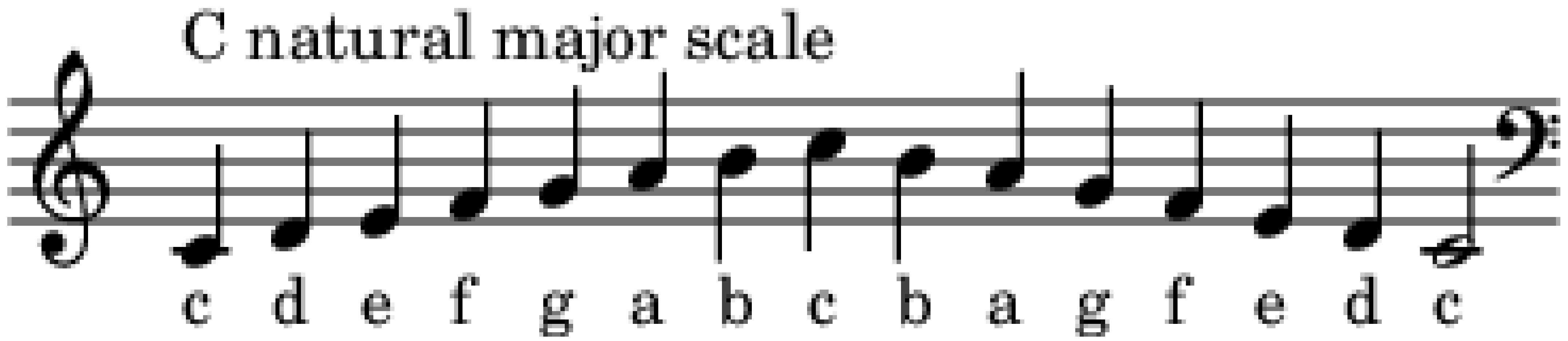

C natural major scale
c d e f g a b c b a g f e d c

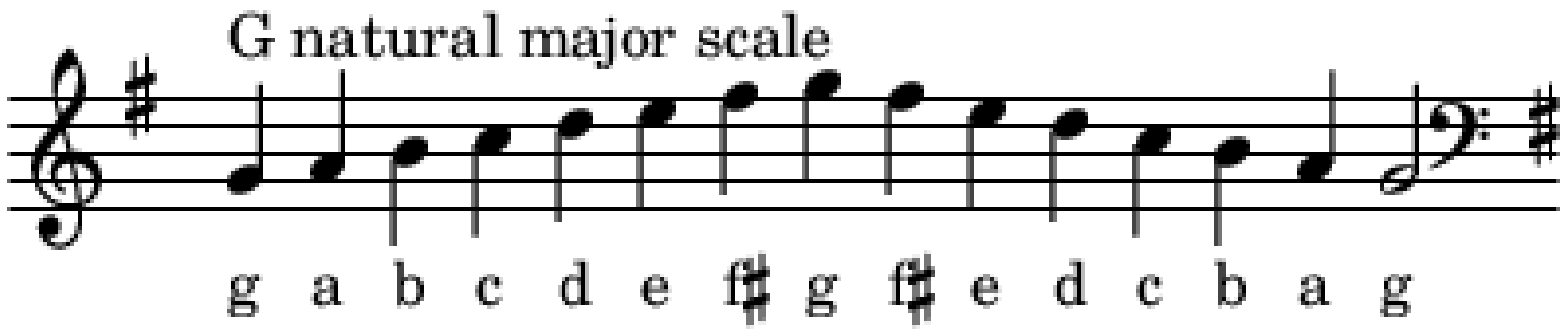


D♭ E♭ G♭ A♭ B♭ D♭ E♭ G♭ A♭ B♭
C♯ D♯ F♯ G♯ A♯ C♯ D♯ F♯ G♯ A♯
C D E F G A B C D E F G A B C

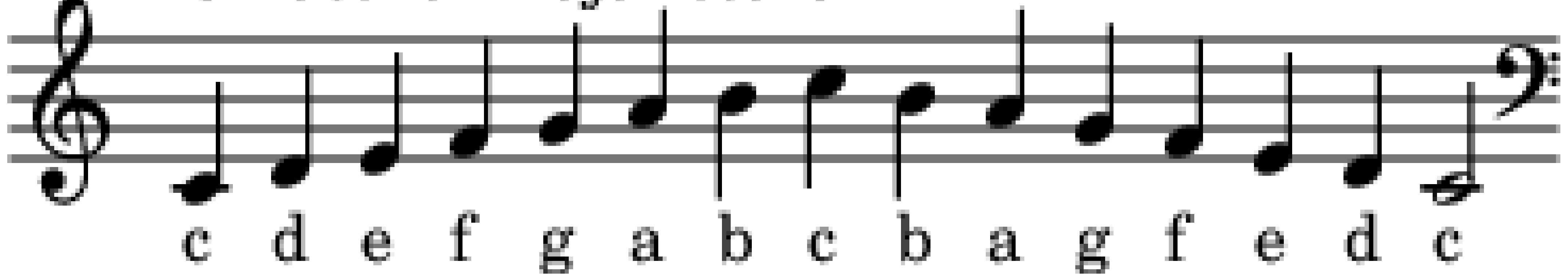


D♭ E♭ G♭ A♭ B♭ D♭ E♭ G♭ A♭ B♭
C C♯ D D♯ E F F♯ G G♯ A A♯ B C C♯ D D♯ E F F♯ G G♯ A A♯ B C

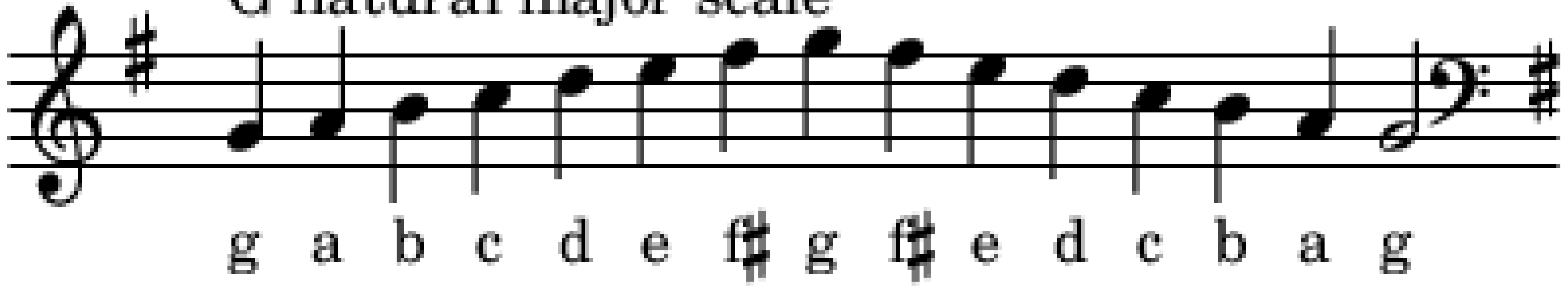

C natural major scale
c d e f g a b c b a g f e d c


G natural major scale
g a b c d e f♯ g f♯ e d c b a g

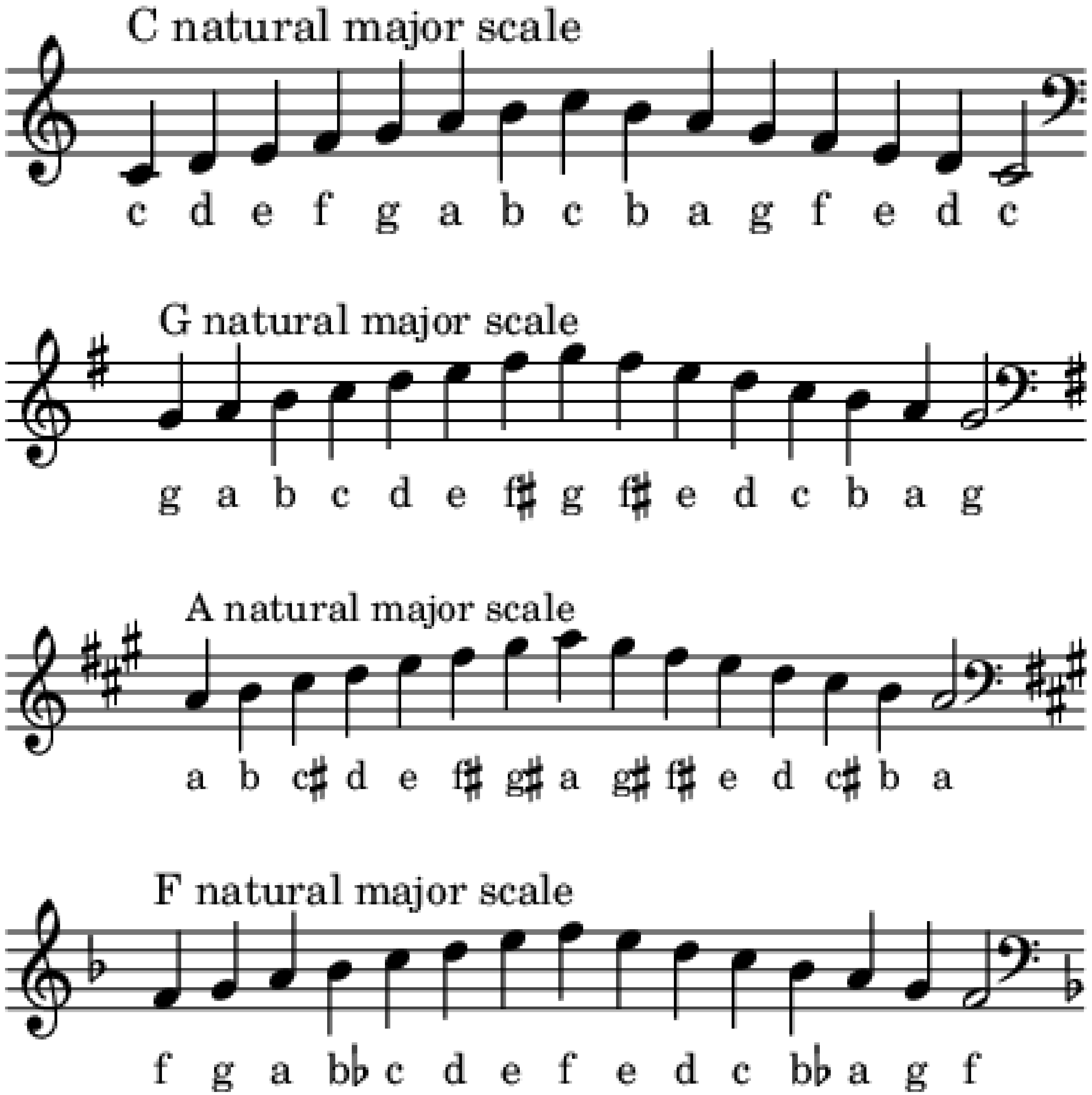
C natural major scale



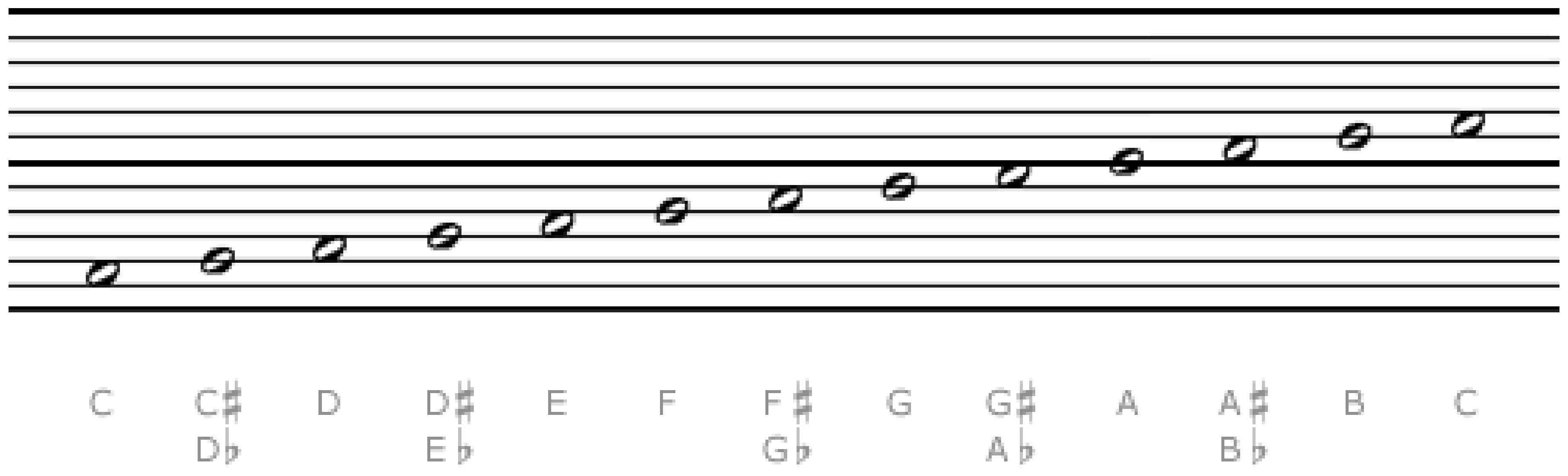
G natural major scale



A natural major scale

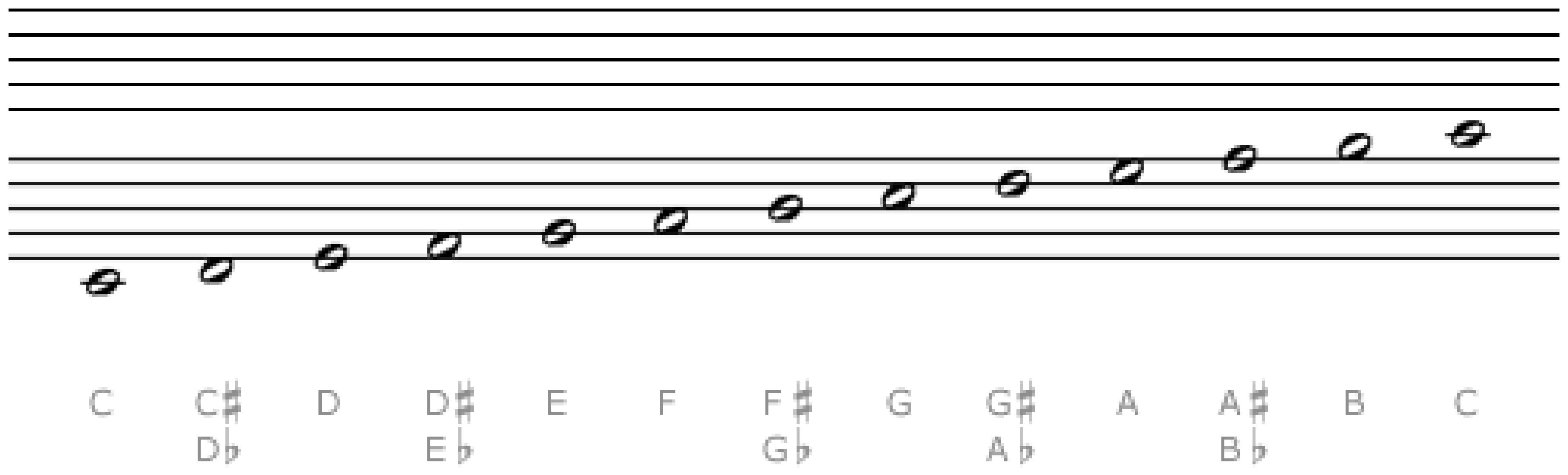

C natural major scale
c d e f g a b c b a g f e d c
G natural major scale
g a b c d e f♯ g f♯ e d c b a g
A natural major scale
a b c♯ d e f♯ g♯ a g♯ f♯ e d c♯ b a
F natural major scale
f g a b♭ c d e f e d c b♭ a g f

1860s and 1870s



Notation for the System of Equal Tones by Gustave Decher, 1877

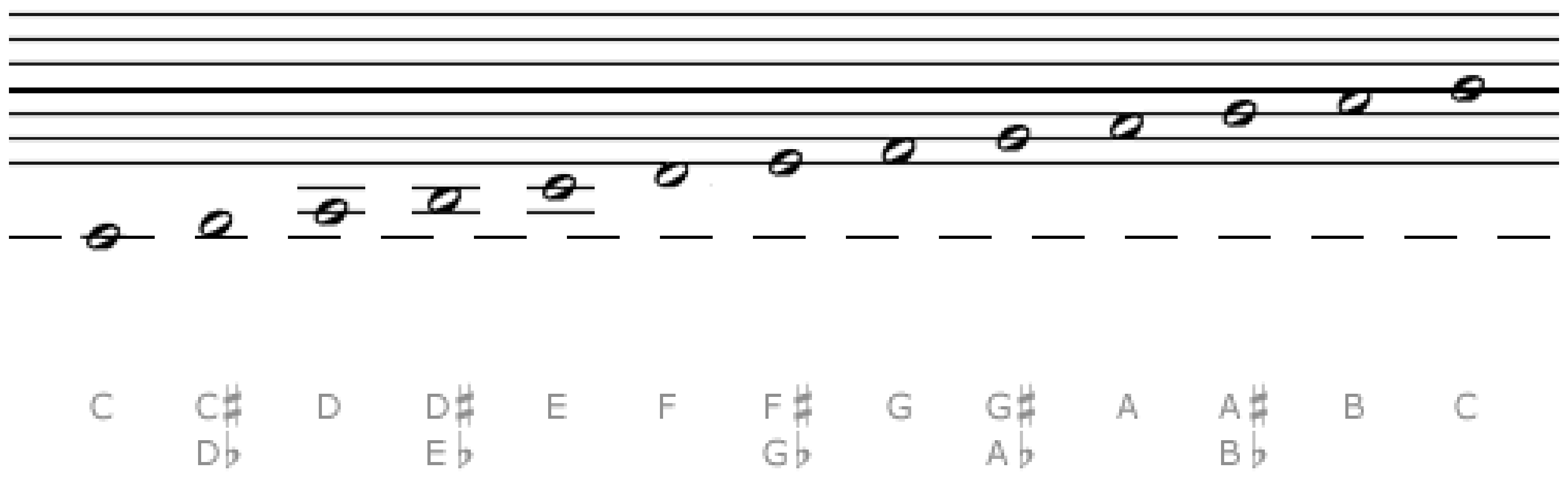
1840s and 1850s



Untitled by Heinrich Richter, 1847

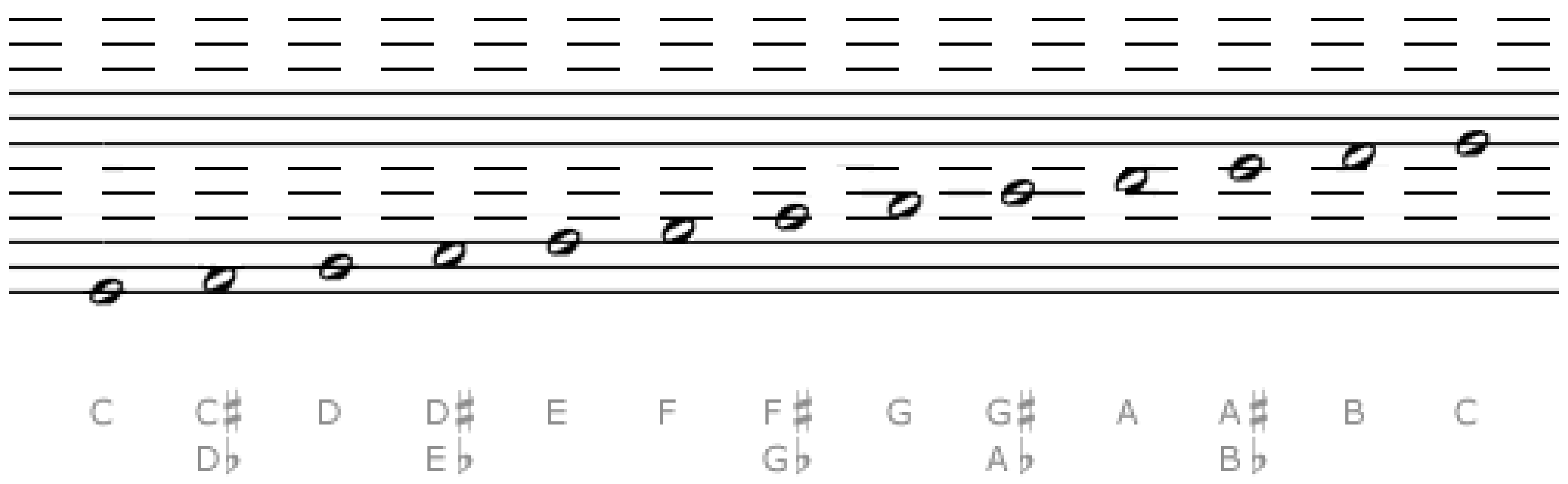
https://musicnotation.org/

1900s and 1910s



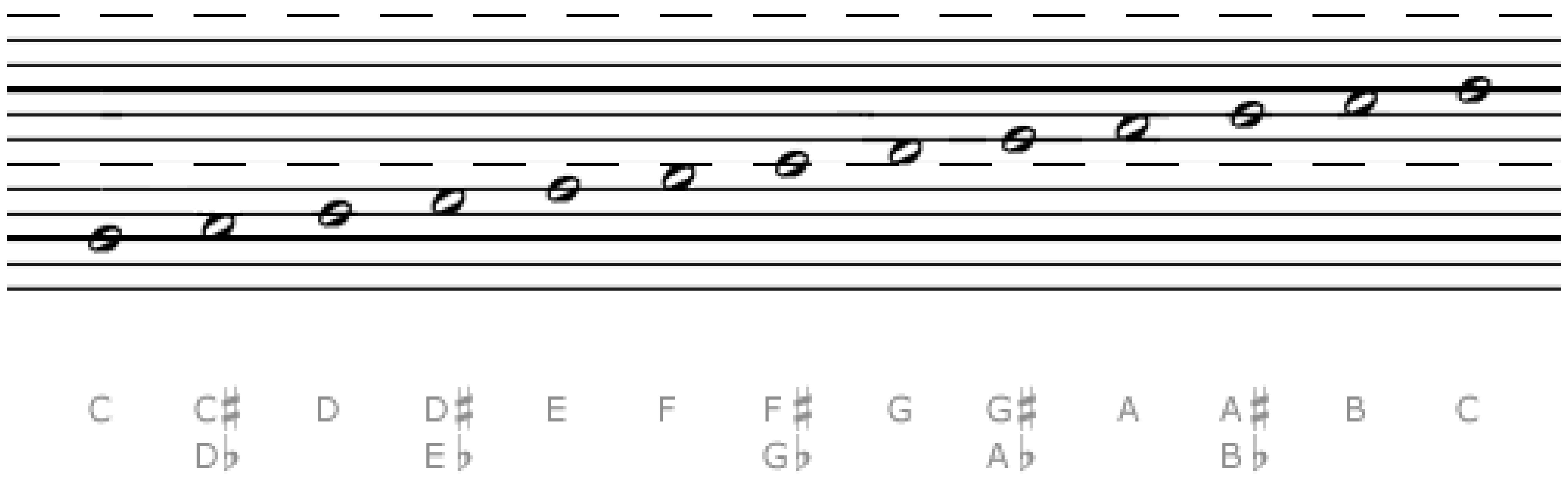
Note for Note by Walter H. Thelwall, 1897

1880s and 1890s

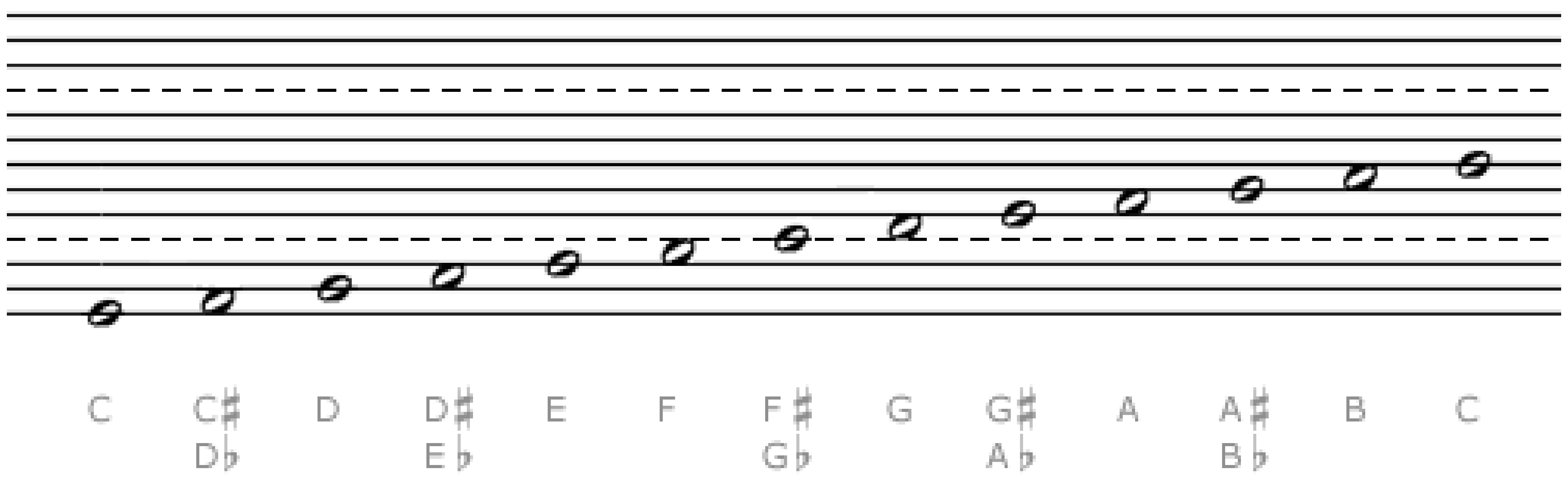


Ambros System by August Ambros, 1883

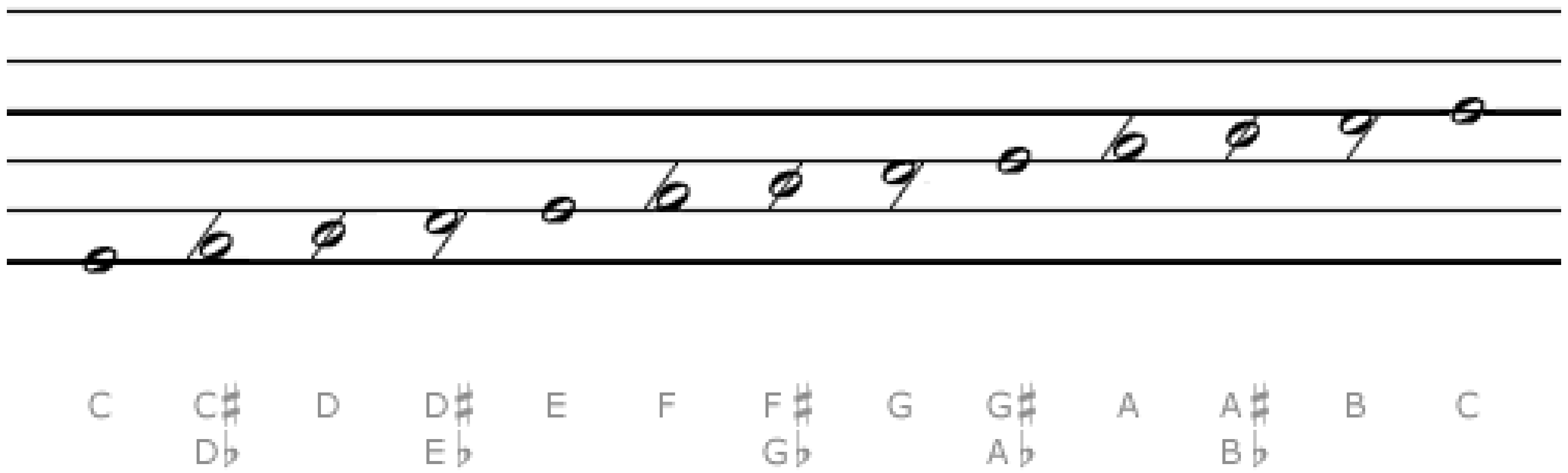
1920s and 1930s



Douzave System by John Leon Acheson, 1936

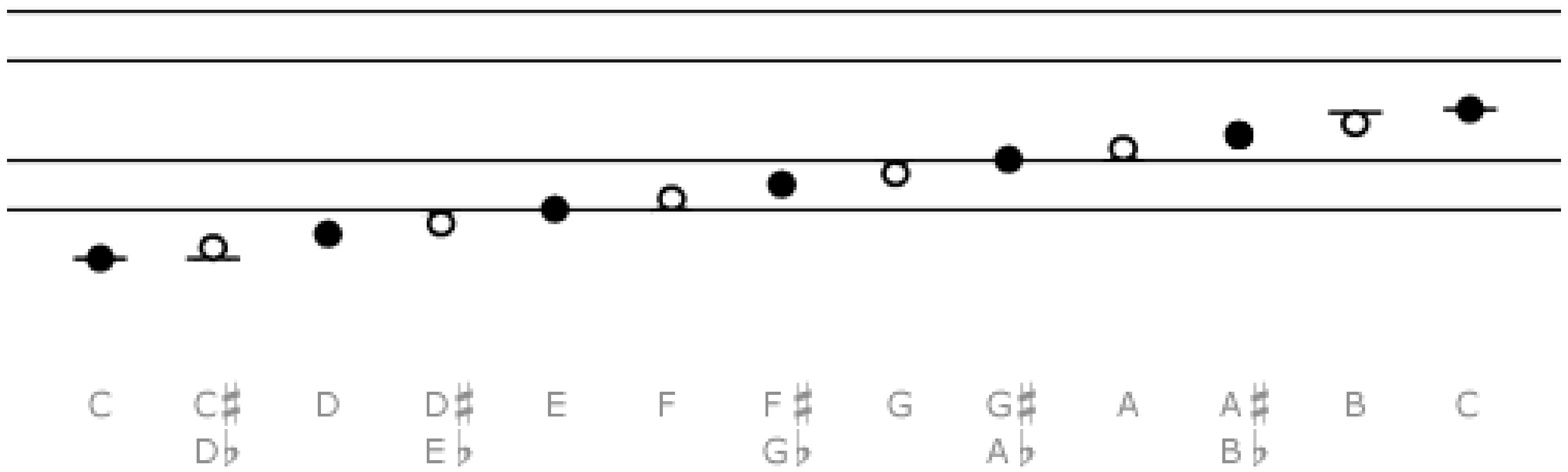


Notagraph by Constance Virtue, 1933

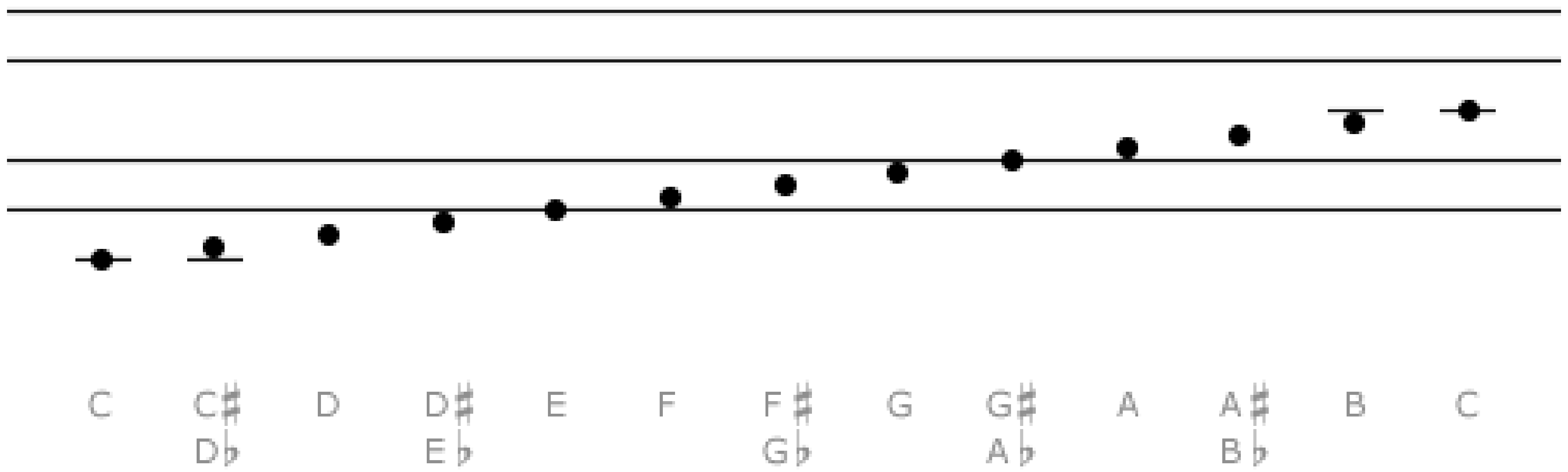


Untitled by Arnold Schoenberg, 1924

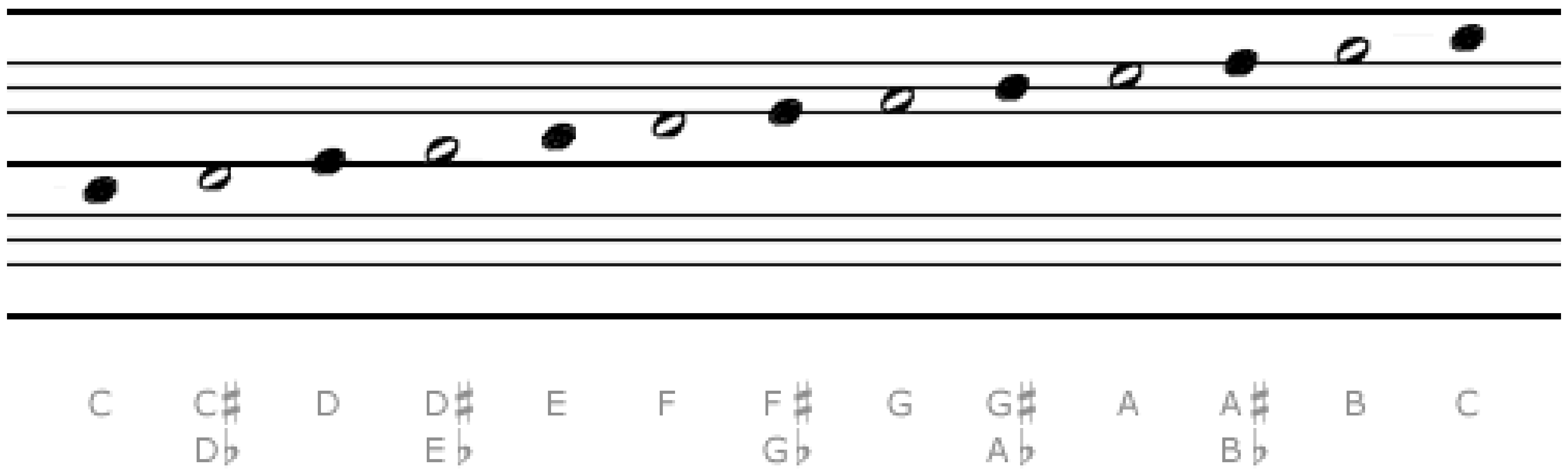
1940s and 1950s



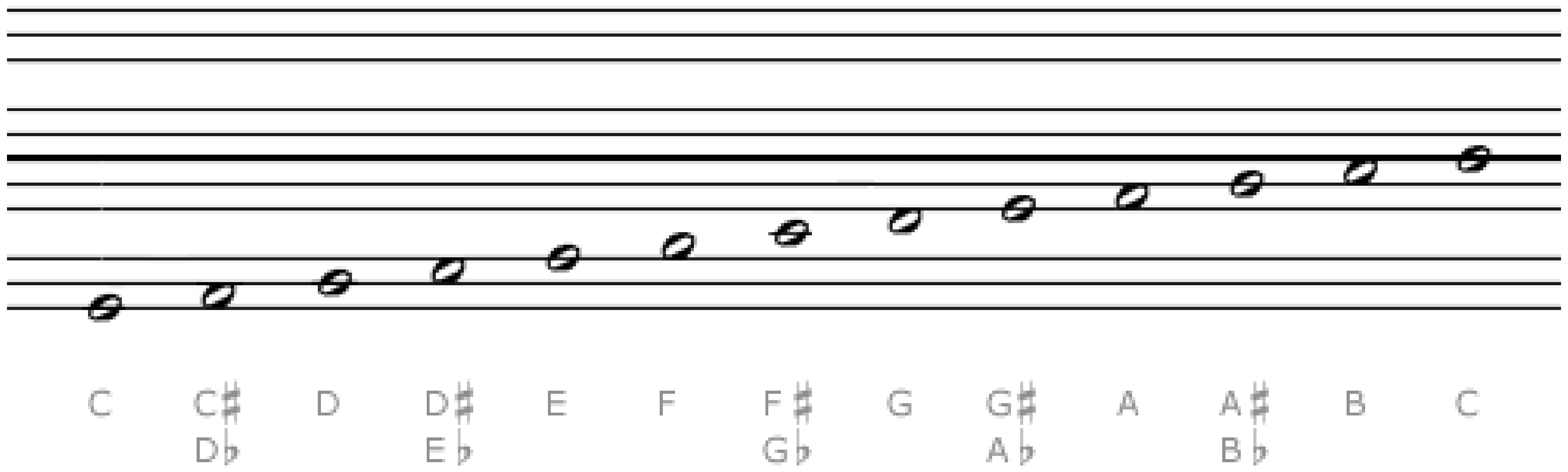
Untitled by Johannes Beyreuther, 1959



Panot Notation by George Skapski, 1956



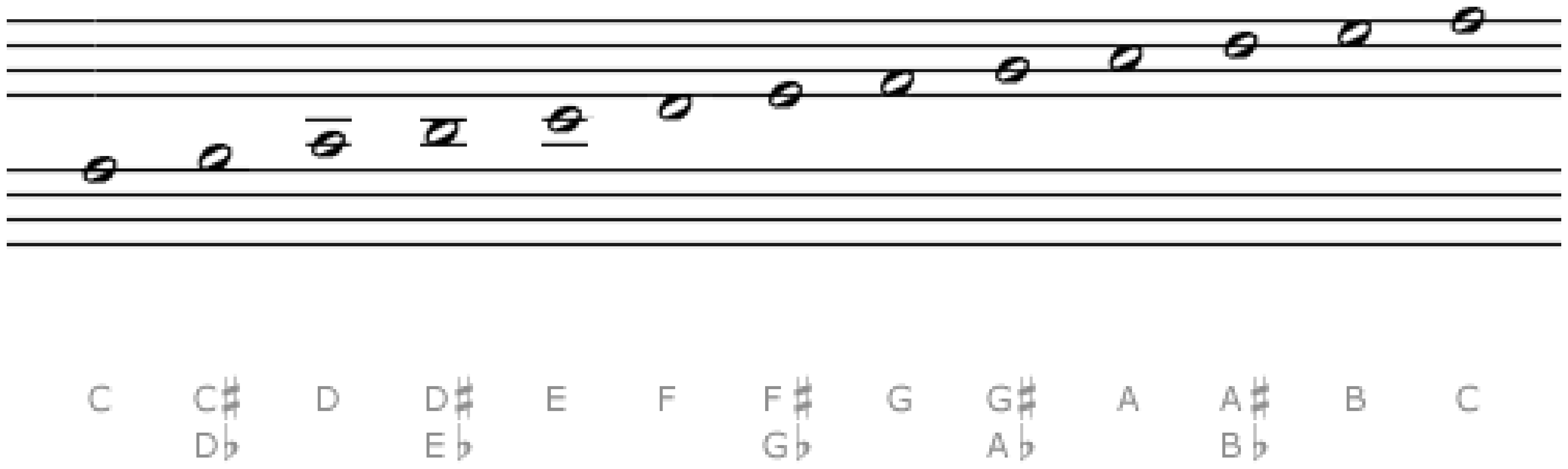
Isomorph Notation by Tadeusz Wójcik, 1952



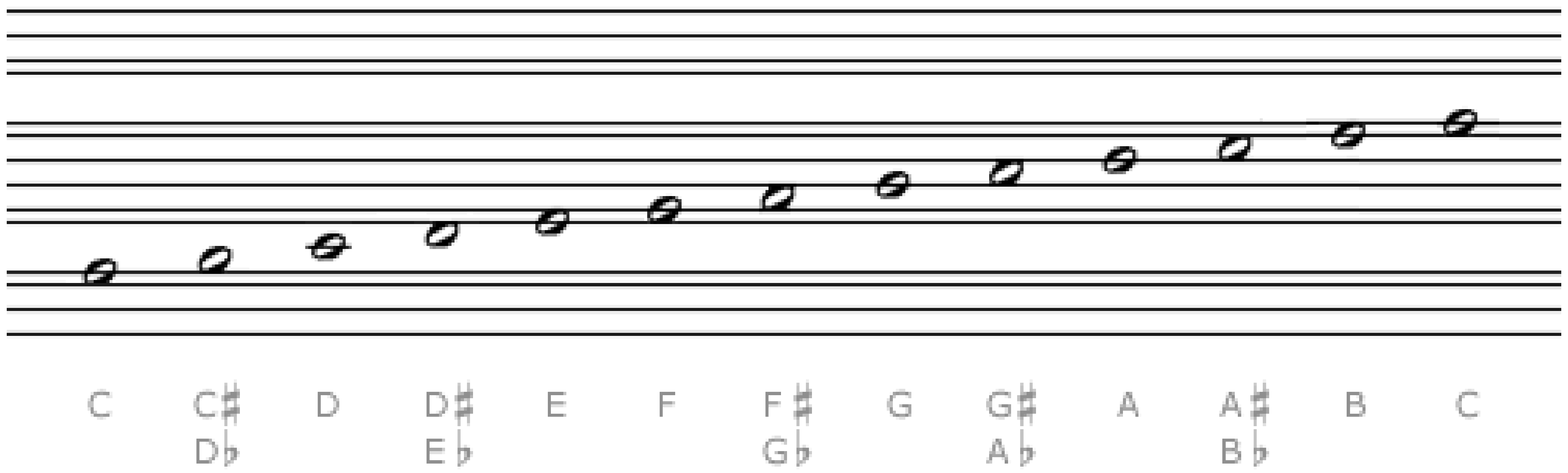
Notation Godjevatz by Velizar Godjevatz, 1948

1960s and 1970s

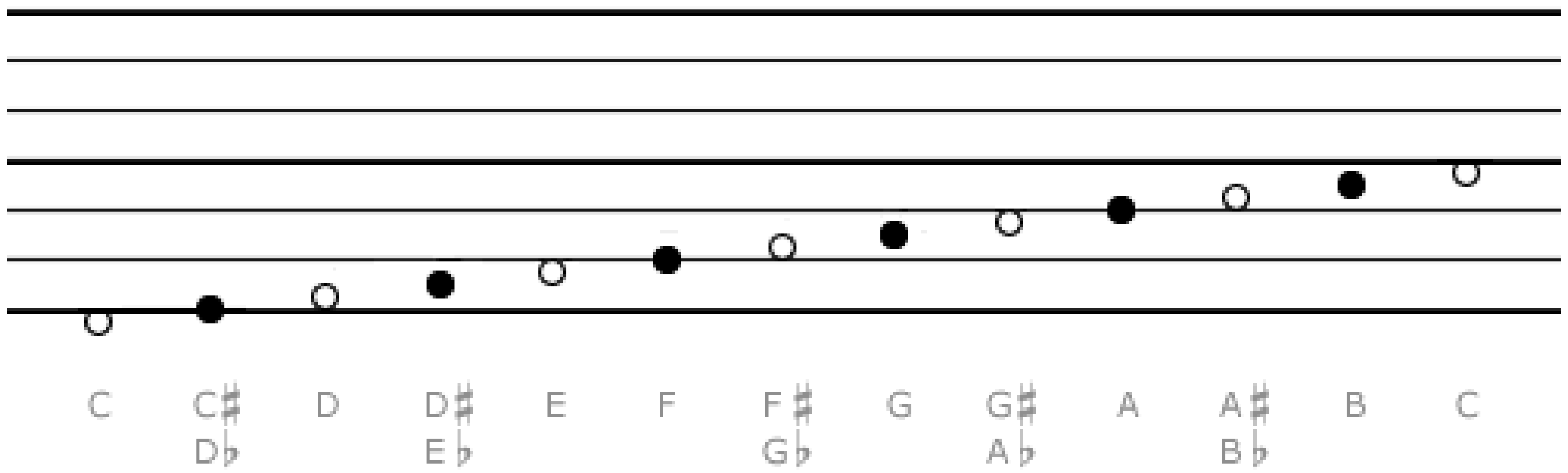
Proportional Chromatic Musical Notation by Henri Carcelle, 1977



A-B Chromatic Notation by Albert Brennink, 1976



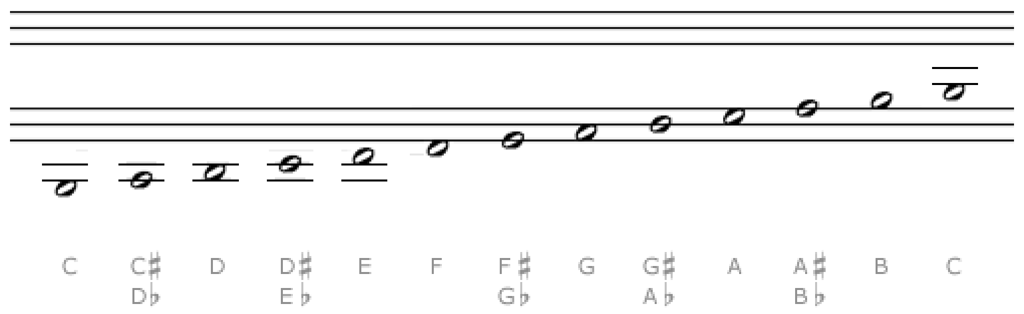
Avique Notation by Anne and Bill Collins, 1974



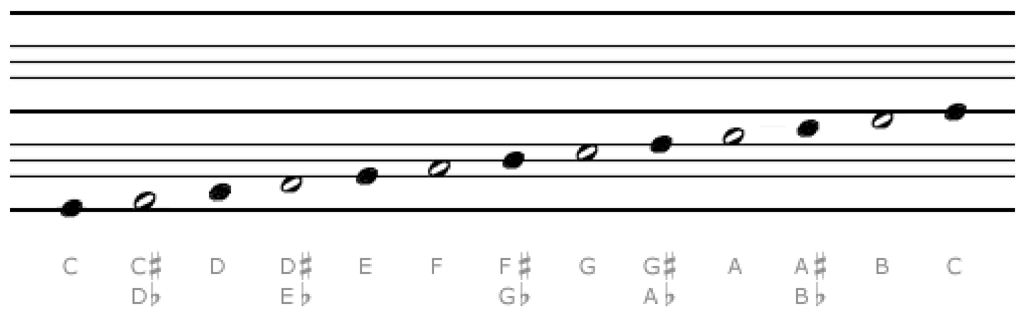
6-6 Klavar by Cornelis Pot, 1972

https://musicnotation.org/

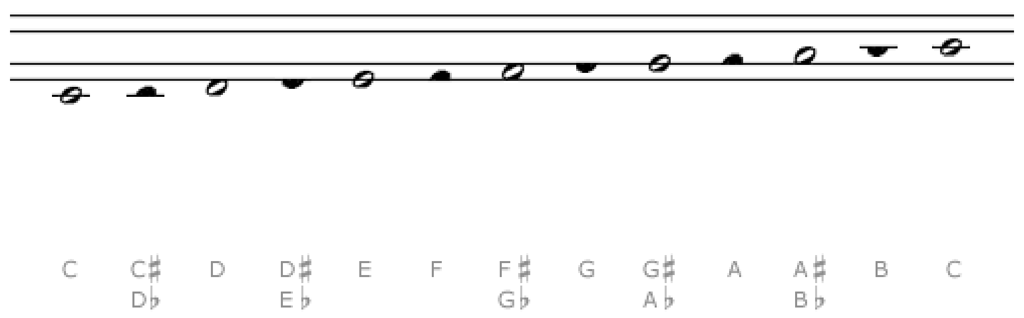
1980s



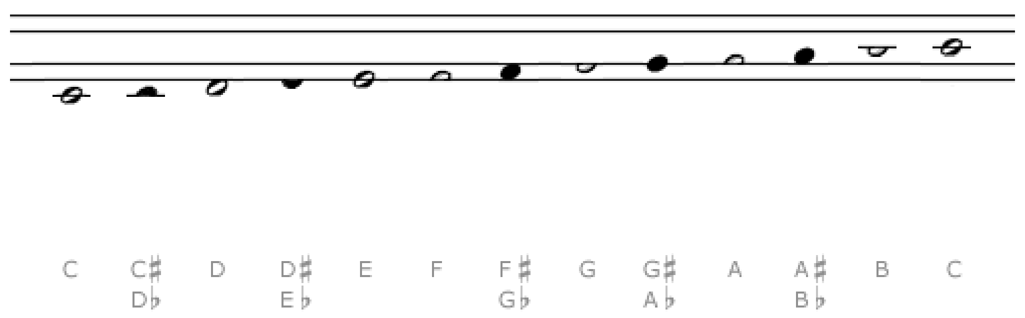
Keyboard (7-5) Trigram Notation by Richard Parncutt, 1989



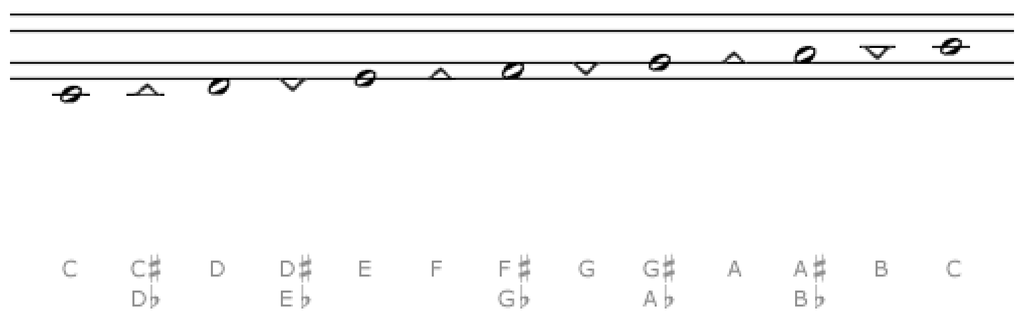
DA Notation by Rich Reed, 1986



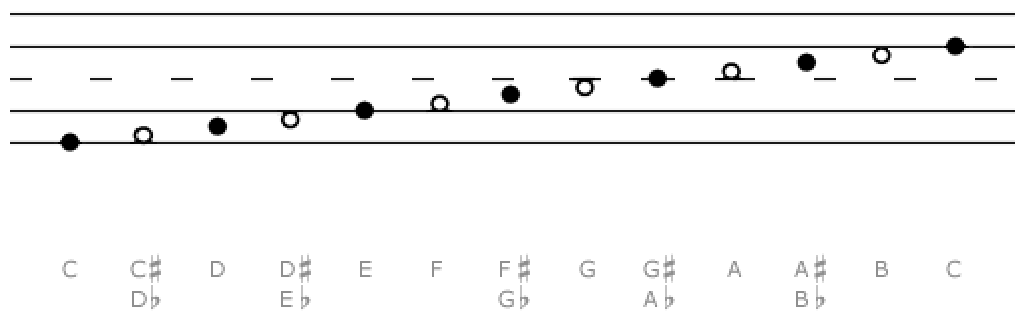
Chromatic Twinline by Leo de Vries, 1986



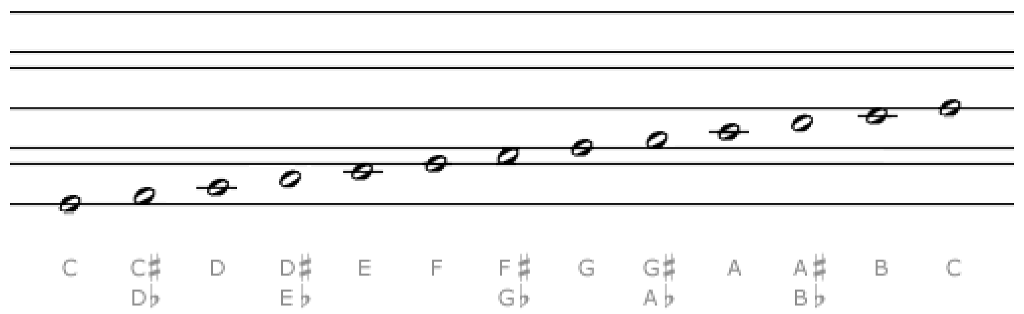
Diatonic Twinline by Leo de Vries, 1986



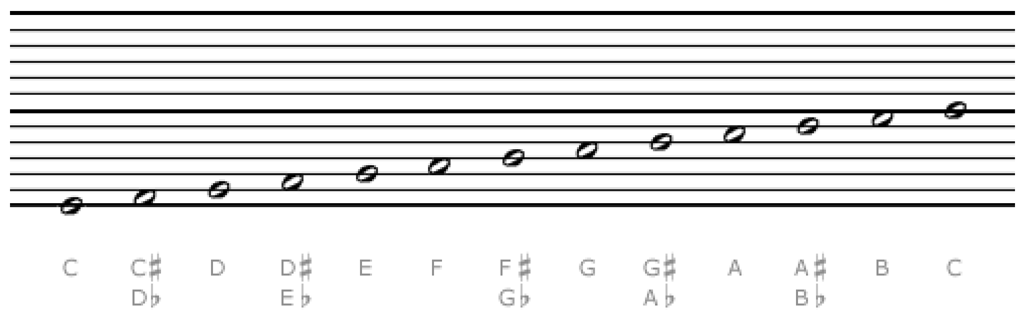
Twinline Notation by Thomas Reed, 1986



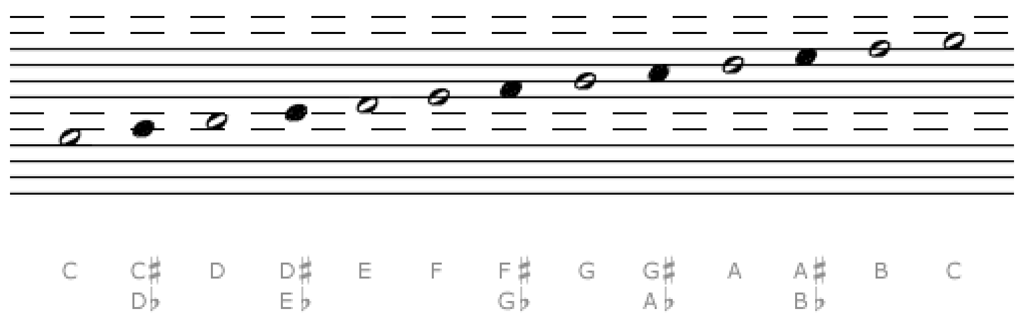
Chromatic 6-6 Notation by Johannes Beyreuther, 1985



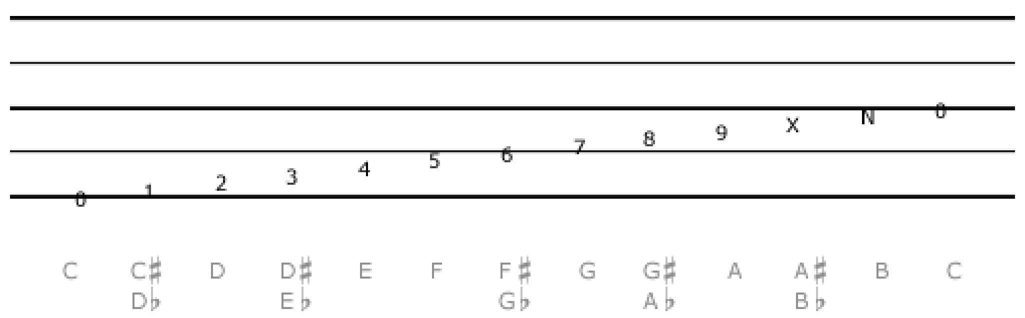
0-5-7 Notation by Richard Parncutt, 1984



Untitled by Klaus Lieber, 1983



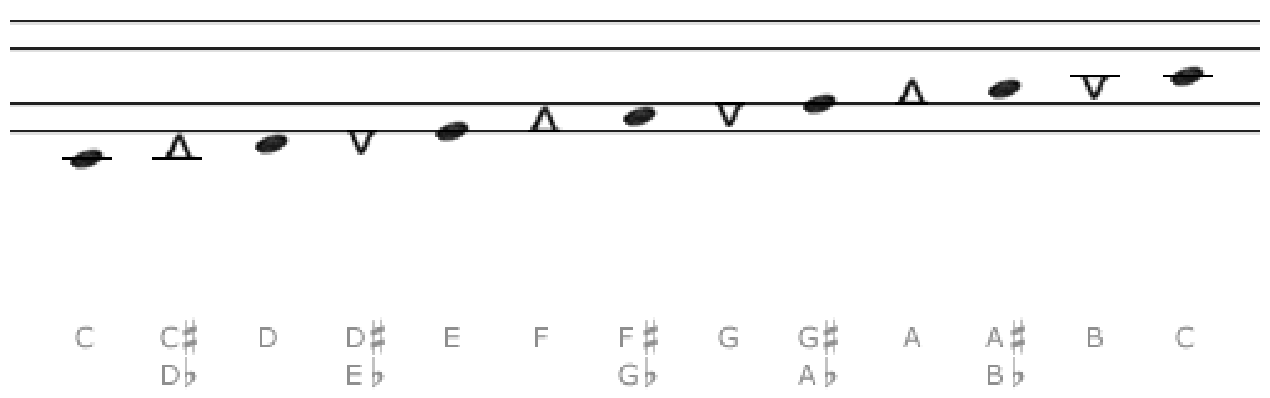
Untitled by Franz Grassl, 1983



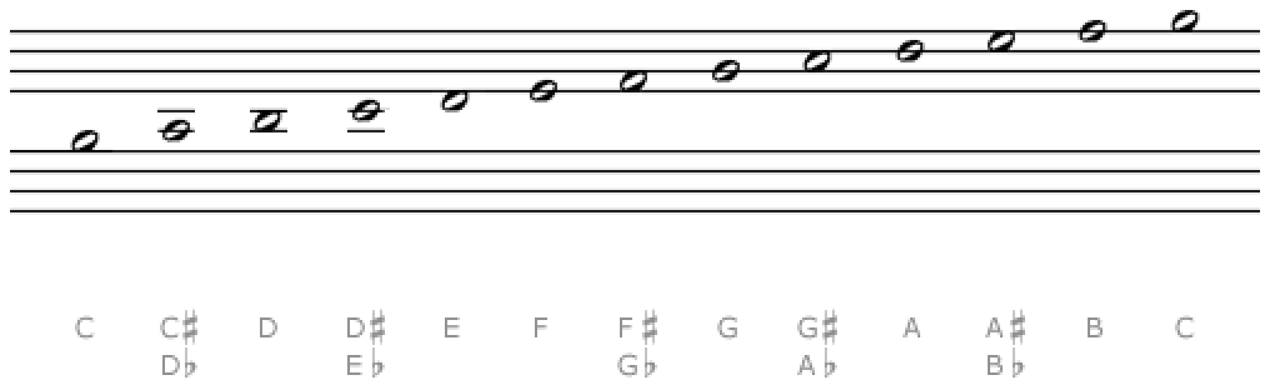
Untitled by Robert Stuckey, 1983

https://musicnotation.org/

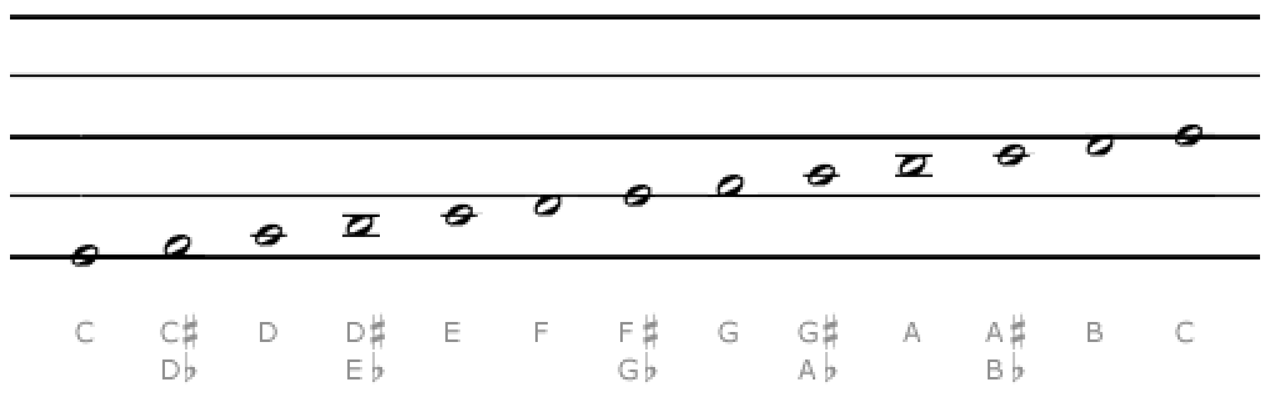
1990s



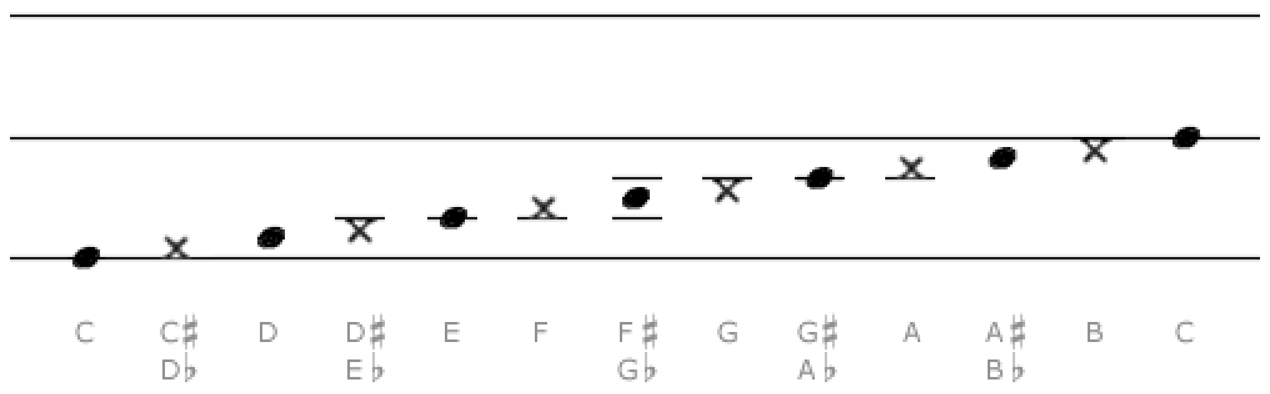
Bilinear Notation by José A. Sotorrío, 1997



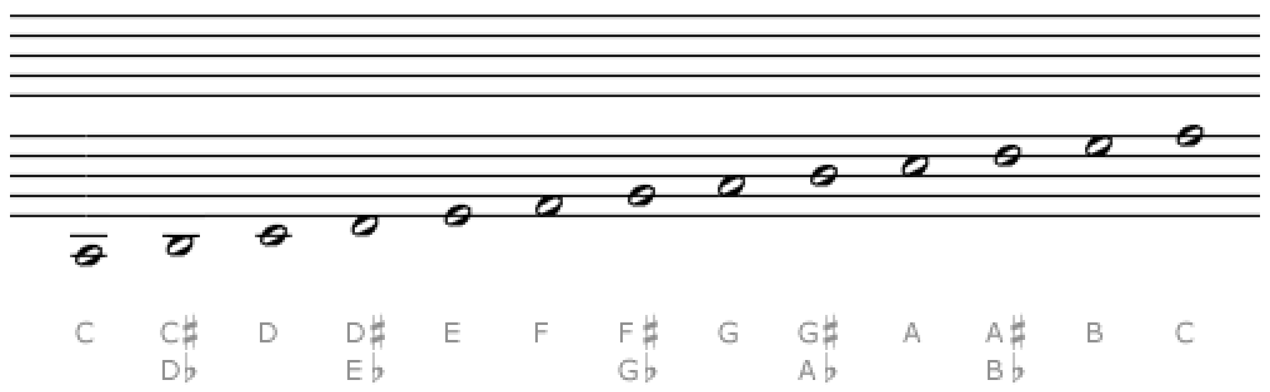
6-6 Tetragram by Richard Parncutt, 1996



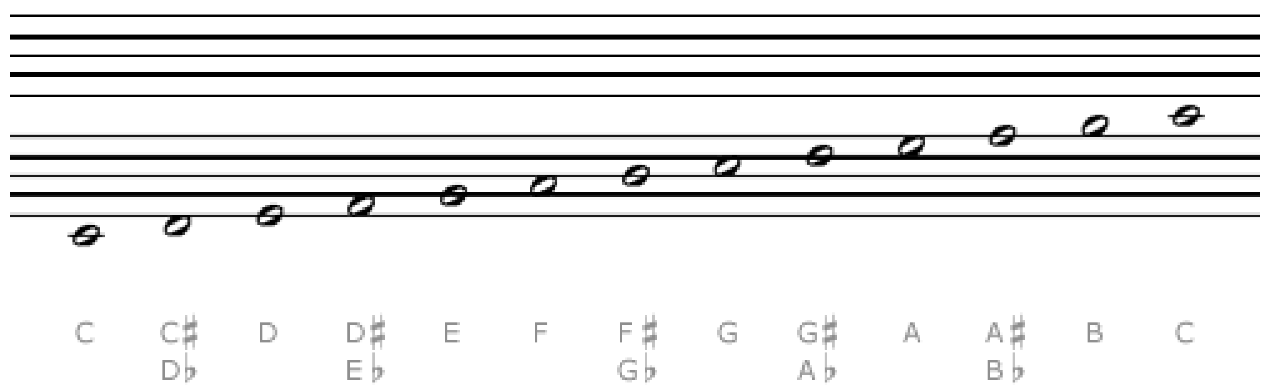
MUTO Notation by MUTO Foundation, 1995



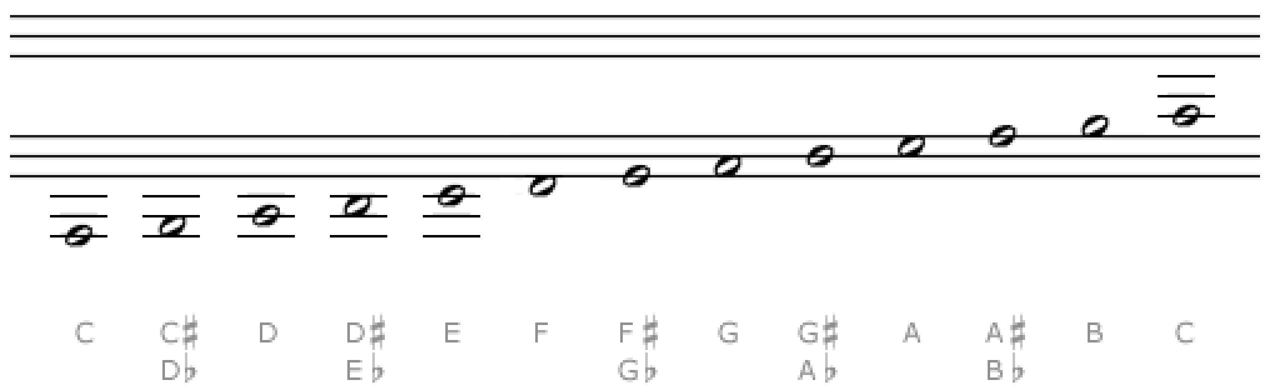
Untitled by Nicolai Dolmatov, 1995



Untitled by Grace Frix, 1992



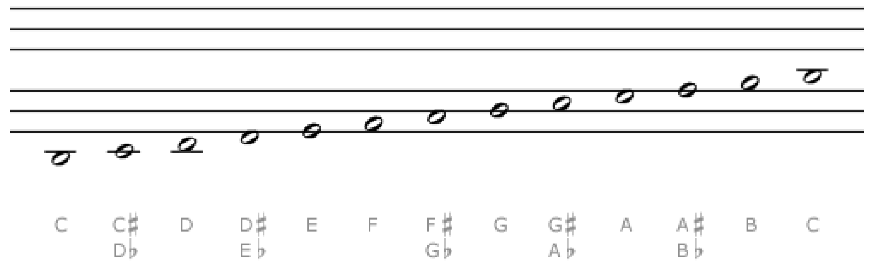
C-Symmetrical Semitone Notation by Ronald Sadlier, 1991



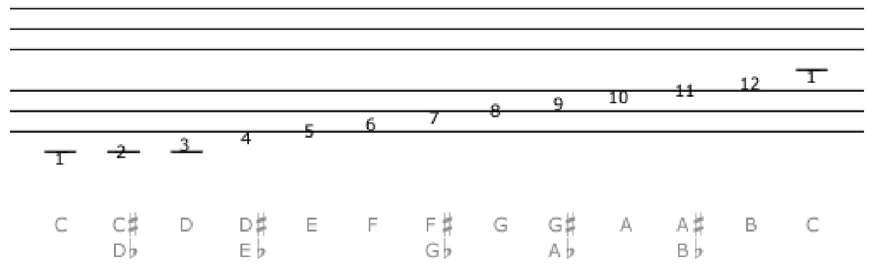
6-6 Trigram Notation by Richard Parncutt, 1990

https://musicnotation.org/

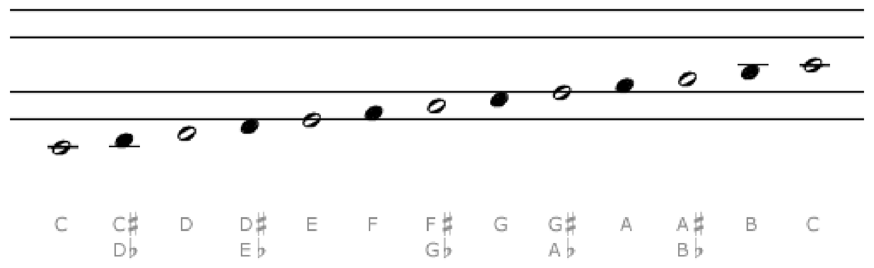
2000s



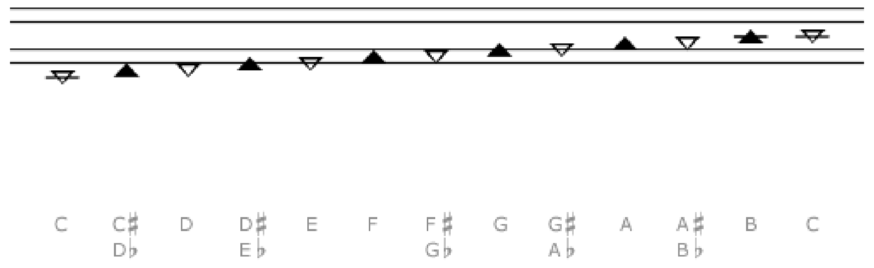
Numbered Notes, Notes-Only by Jason MacCoy, 2009



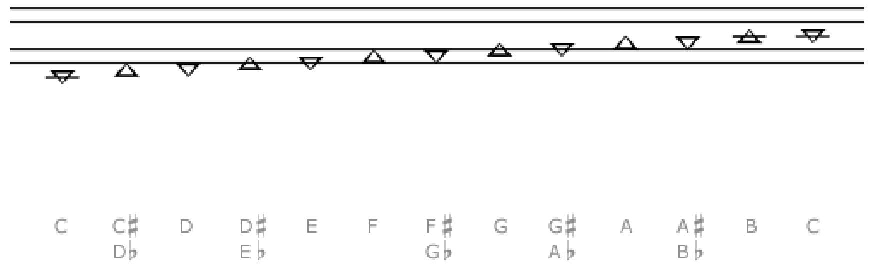
Numbered Notes, Numbers-Only by Jason MacCoy, 2009



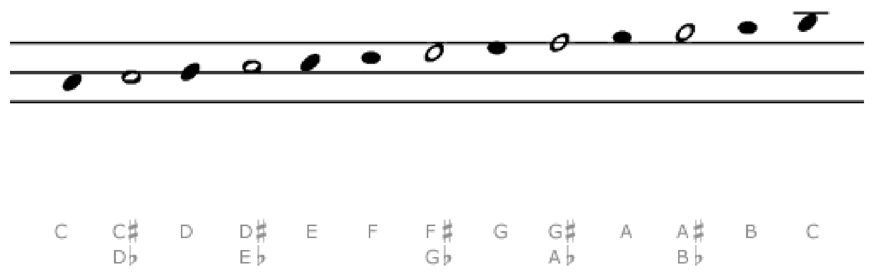
Chromatic Lyre Notation by Jan Braunstein, 2009



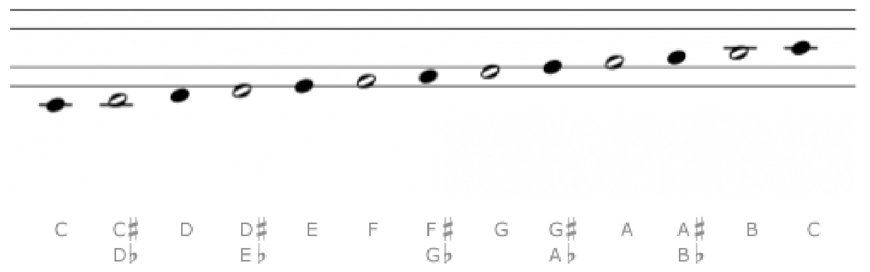
TwinNote by Paul Morris, 2009



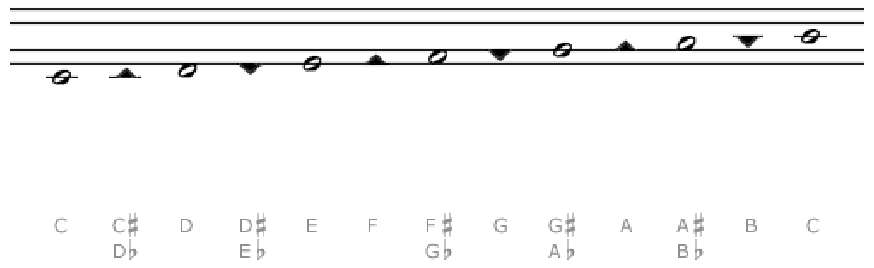
TwinNote TD by Paul Morris, 2009



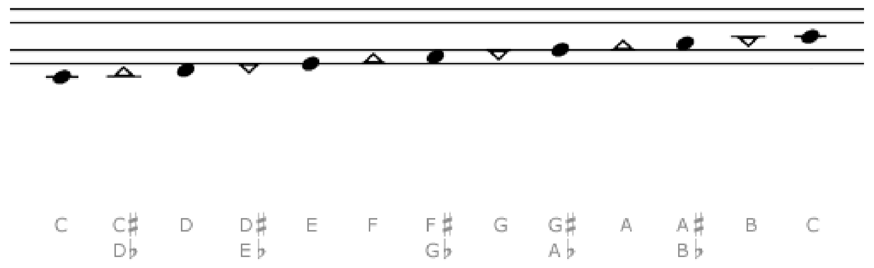
Express Stave, 6-6 Jazz Font by John Keller, 2009



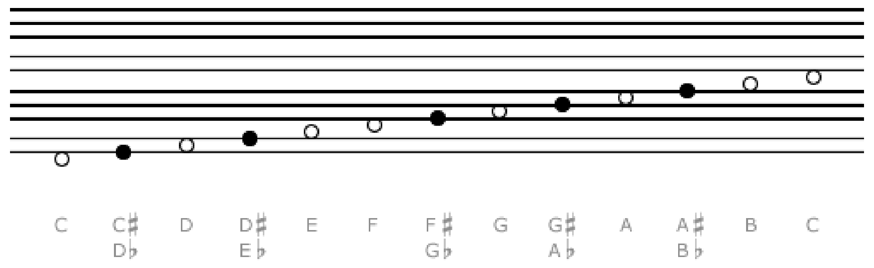
Clairnote DN by Paul Morris, 2006



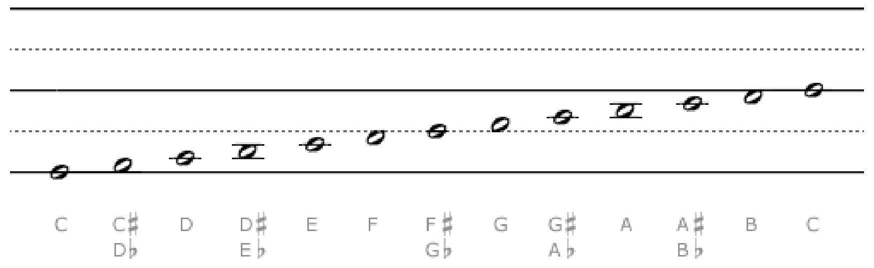
Black Triangle Twinline by Doug Keislar, 2006



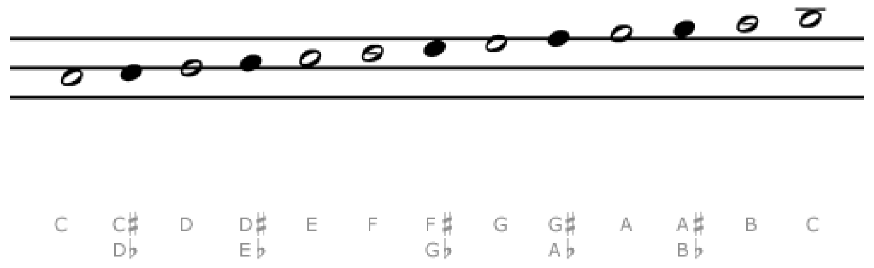
Black-Oval Twinline by Paul Morris, 2006



Klavar, Mirck Version by Jean de Buur, 2006



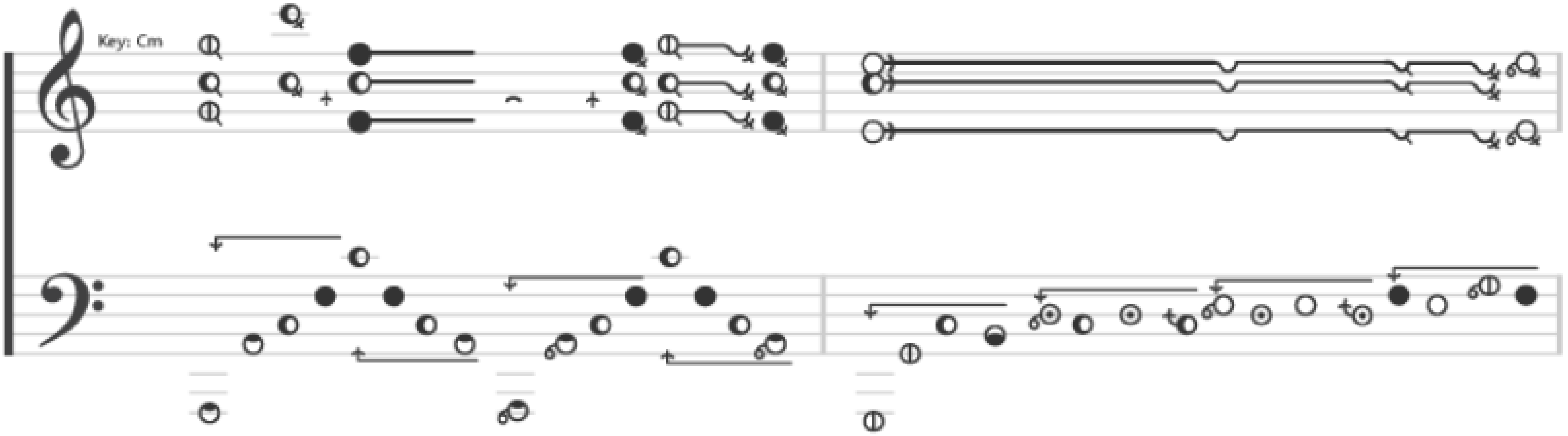
Thumline Notation by Jim Plamondon, 2005

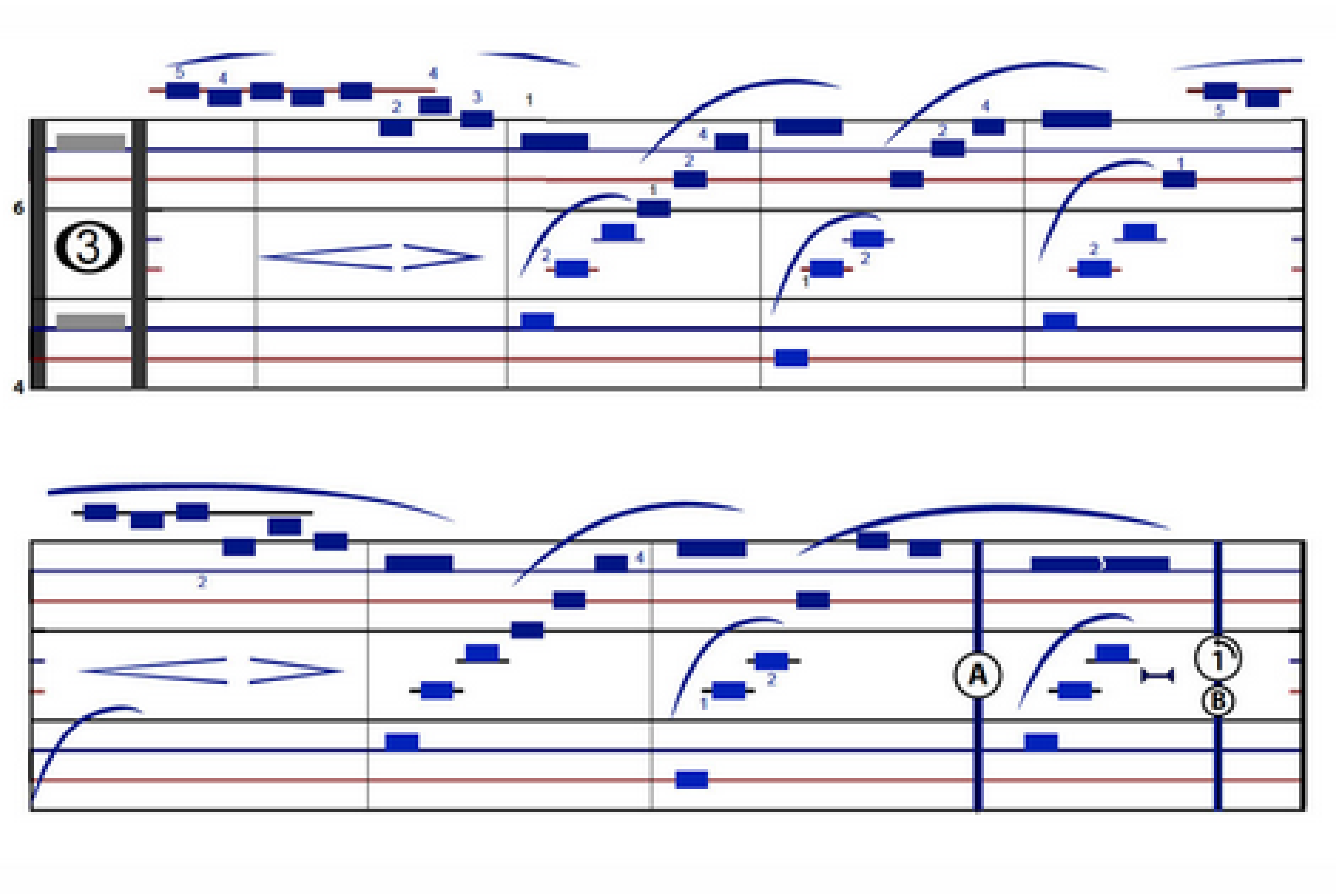


Express Stave, Original Version by John Keller, 2005

Hummingbird

A fresh take on music notation — easier to learn,
faster to read, and simpler for even the trickiest music.





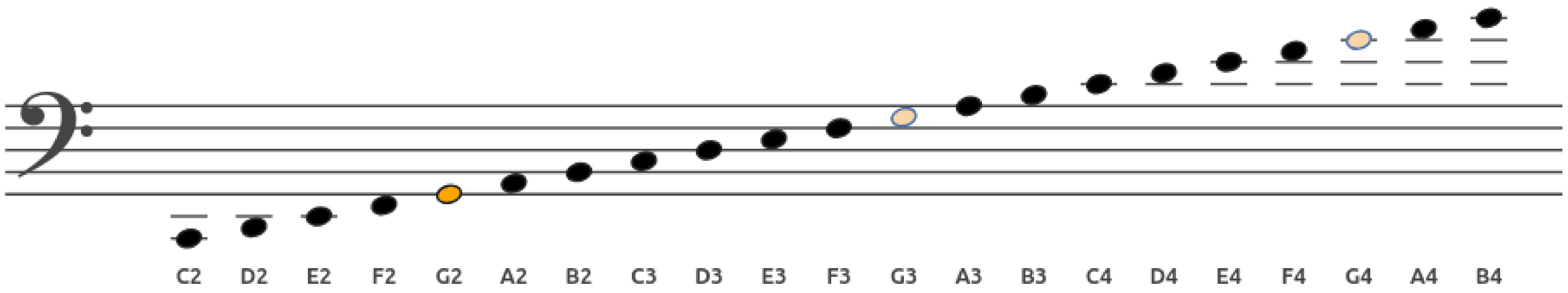
(Excerpt of Beethoven Für Elise written in Dodeka Notation)

DEMO

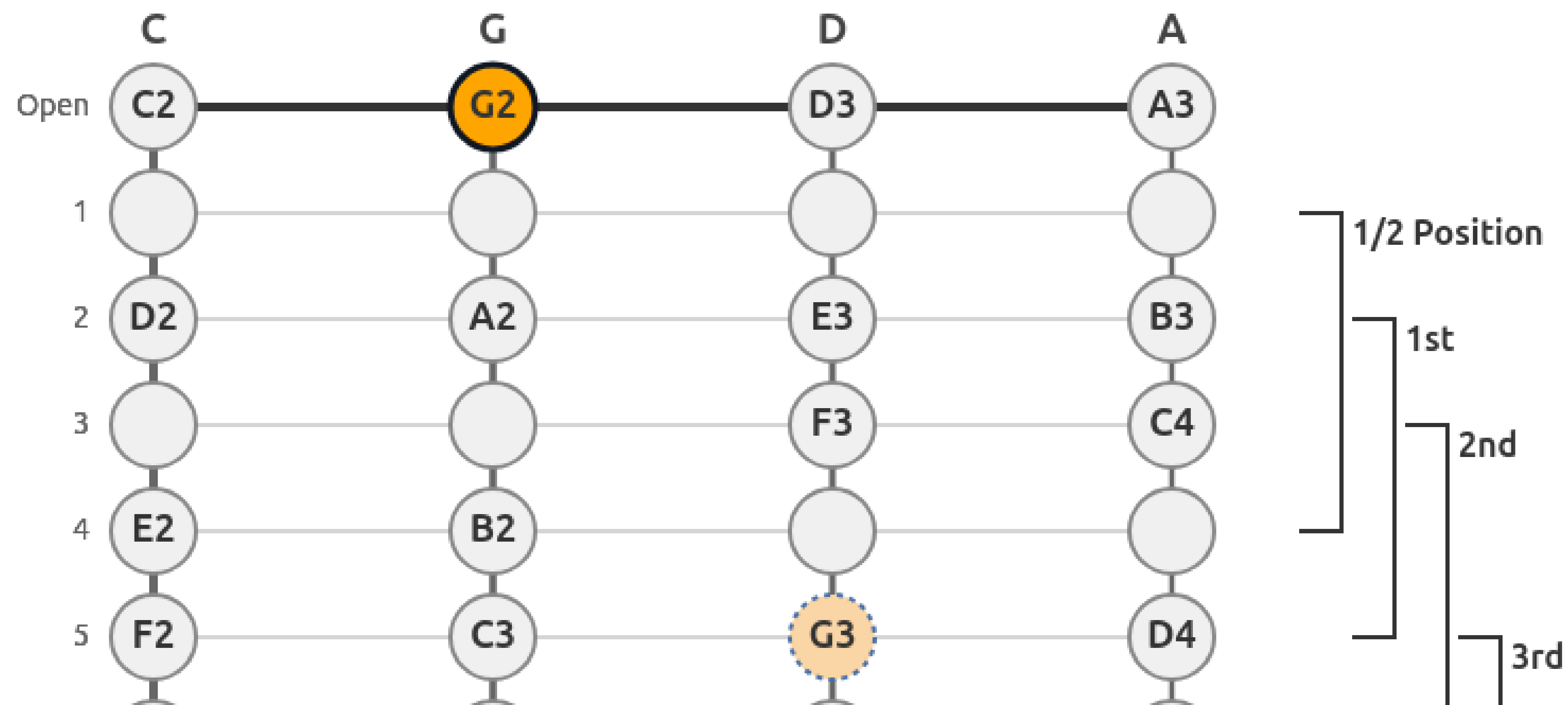
strudel

🎵 Bass Clef Note Selector

☐ C♯ ☐ D♯ ☐ F♯ ☐ G♯ ☐ A♯ ☐ D♭ ☐ E♭ ☐ G♭ ☐ A♭ ☐ B♭



🎻 Cello Fingerboard



Fix finger positions #2

Merged **spirali** merged 2 commits into `spirali:main` from `ttilberg:fix-finger-positions` on Oct 6

Conversation 2 | Commits 2 | Checks 0 | Files changed 1

Changes from **all commits** ▾ **File filter** ▾ **Conversations** ▾ **Jump to** ▾

13 `index.html`

```diff
@@ -362,11 +362,14 @@ <h2>🎻 Cello Fingerboard</h2>

         // Draw position brackets
         const positions = [
-          { name: 'Base Position', startFret: 2 },
-          { name: '2nd Position', startFret: 4 },
-          { name: '3rd Position', startFret: 6 },
-          { name: '4th Position', startFret: 8 },
-          { name: '5th Position', startFret: 10 }
+          { name: '1/2 Position', startFret: 1},
+          { name: '1st', startFret: 2 },
+          { name: '2nd', startFret: 3 },
+          { name: '3rd', startFret: 5 },
+          { name: '4th', startFret: 7 },
+          { name: '5th', startFret: 9 },
+          { name: '6th', startFret: 10 },
+          { name: '7th', startFret: 12 }
         ];

         positions.forEach((position, index) => {
```

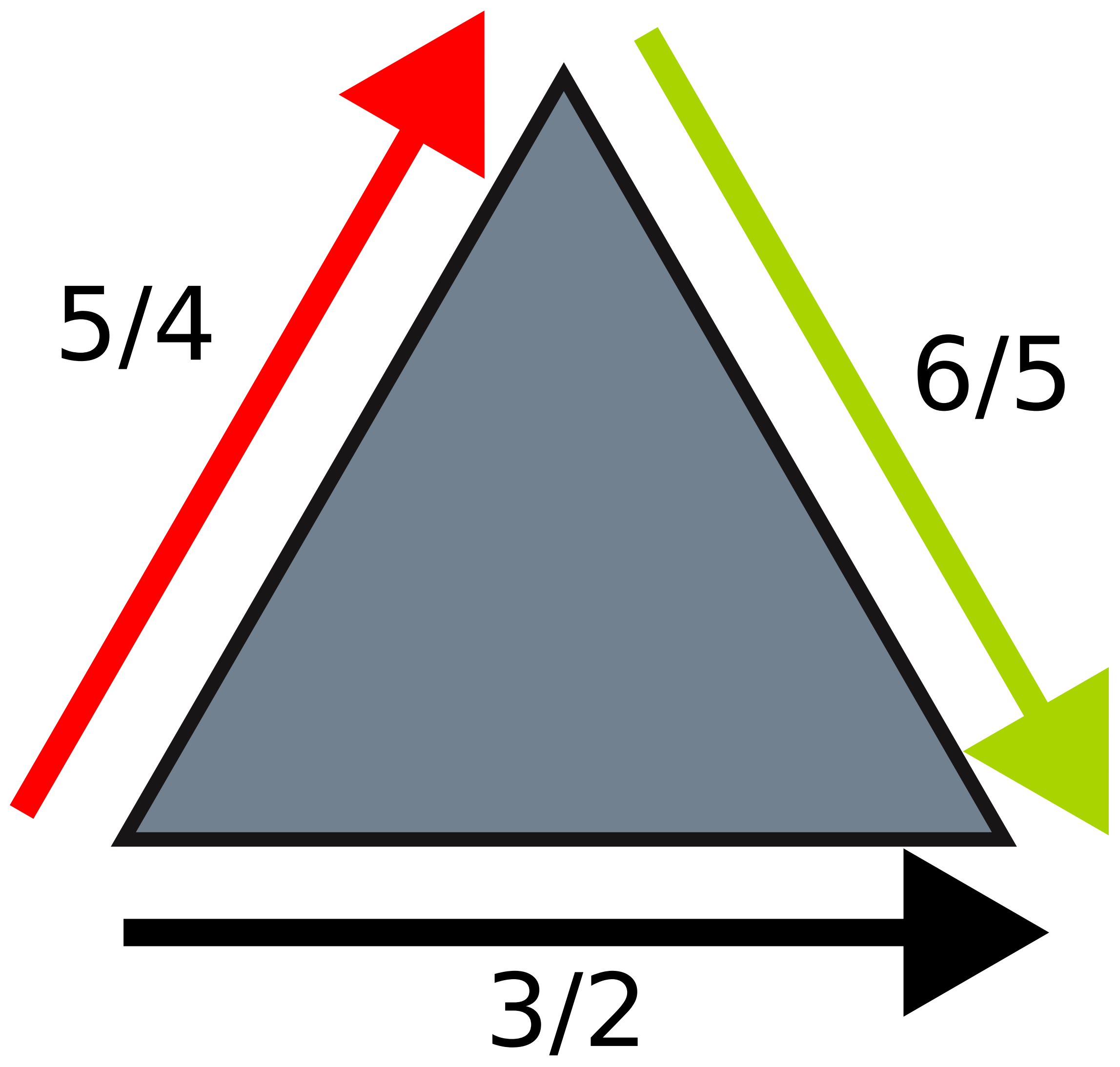
Matematický pohled na hudební teorii

Ada Böhmová

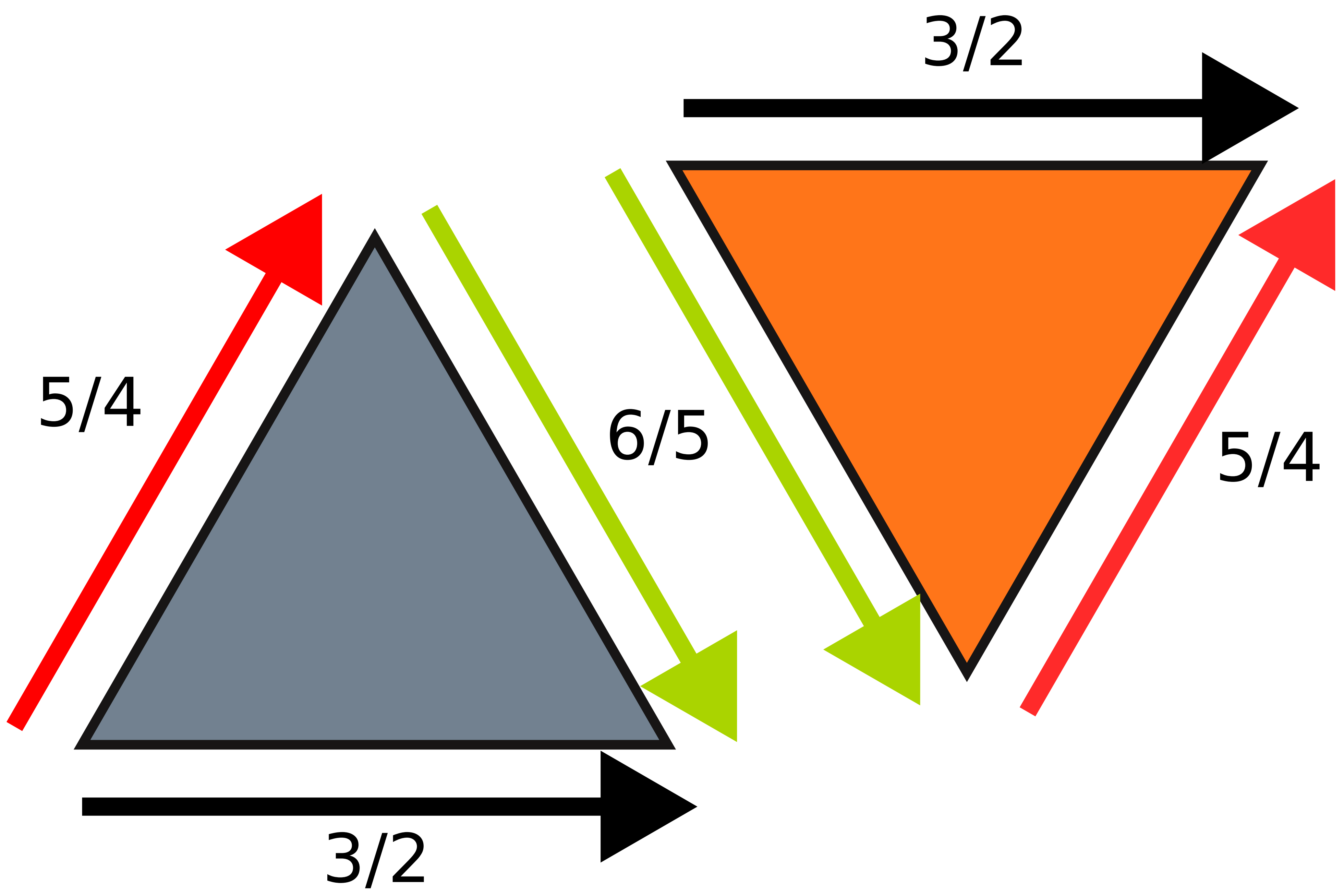
ada@kreatrix.org
github.com/spirali

$$\frac{3}{2} = \frac{6}{5}.\frac{5}{4}$$

$$\frac{3}{2} = \frac{6}{5} \cdot \frac{5}{4}$$



$$\frac{3}{2} = \frac{6}{5} \cdot \frac{5}{4}$$



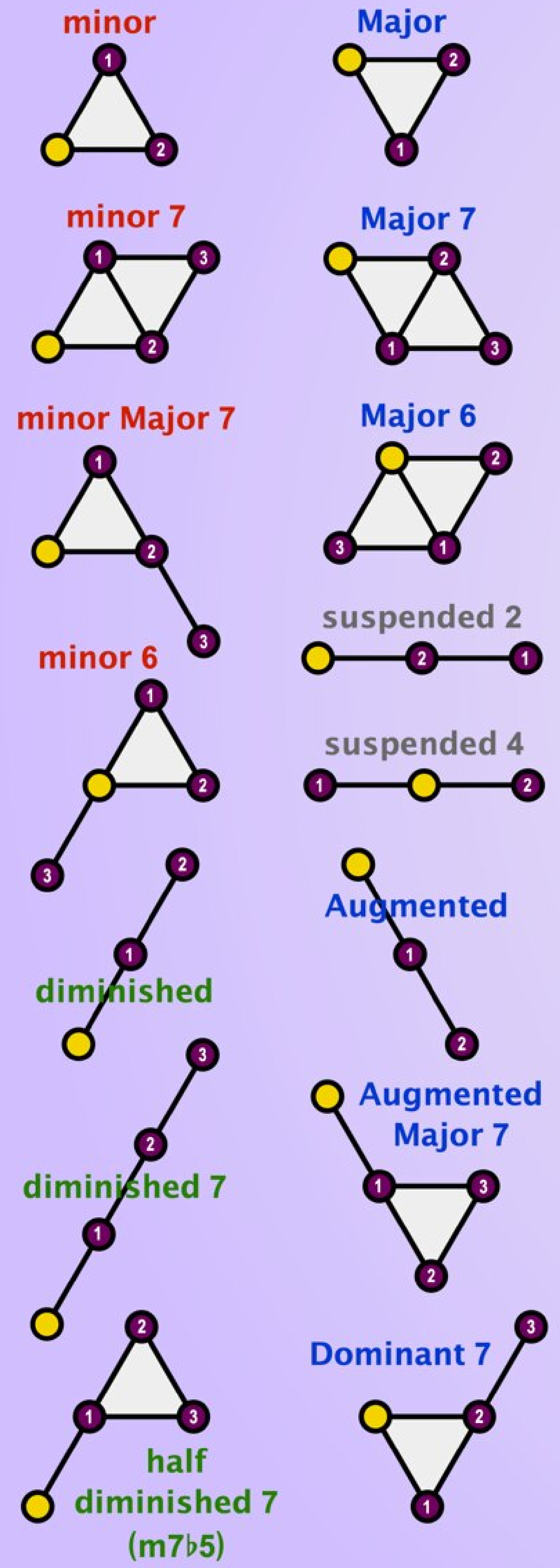
TONNETZ

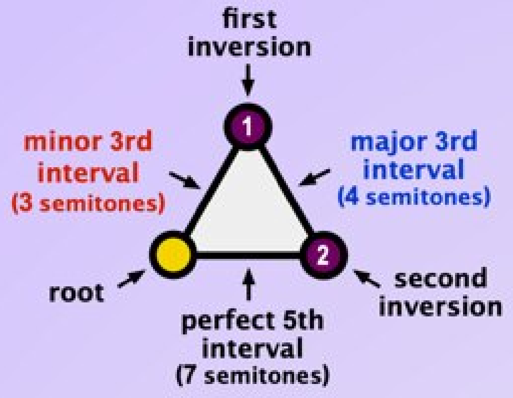
CHORDS

A **chord** is a set of 3 or more **notes** that sound harmonically when played together. Since chords are defined by the **interval** between its notes, in the **Tonnetz** the same type of chord (for example a Major chord) has always the same shape, regardless of where you put the **root note** (Cmaj, Dmaj, Emaj, ...).

The **root note** gives name to the chord and has the lowest pitch, unless the chord is **inverted**. The first inversion of a chord has the same notes, but the note at position 1 (see diagrams below) has the lowest pitch instead. Similarly, the second inversion gets note 2 as its lowest pitch note, etc.

Inverted chords are indicated with the lowest pitch note as part of the name. For example: 2nd inversion of C minor = **Cm/G**.

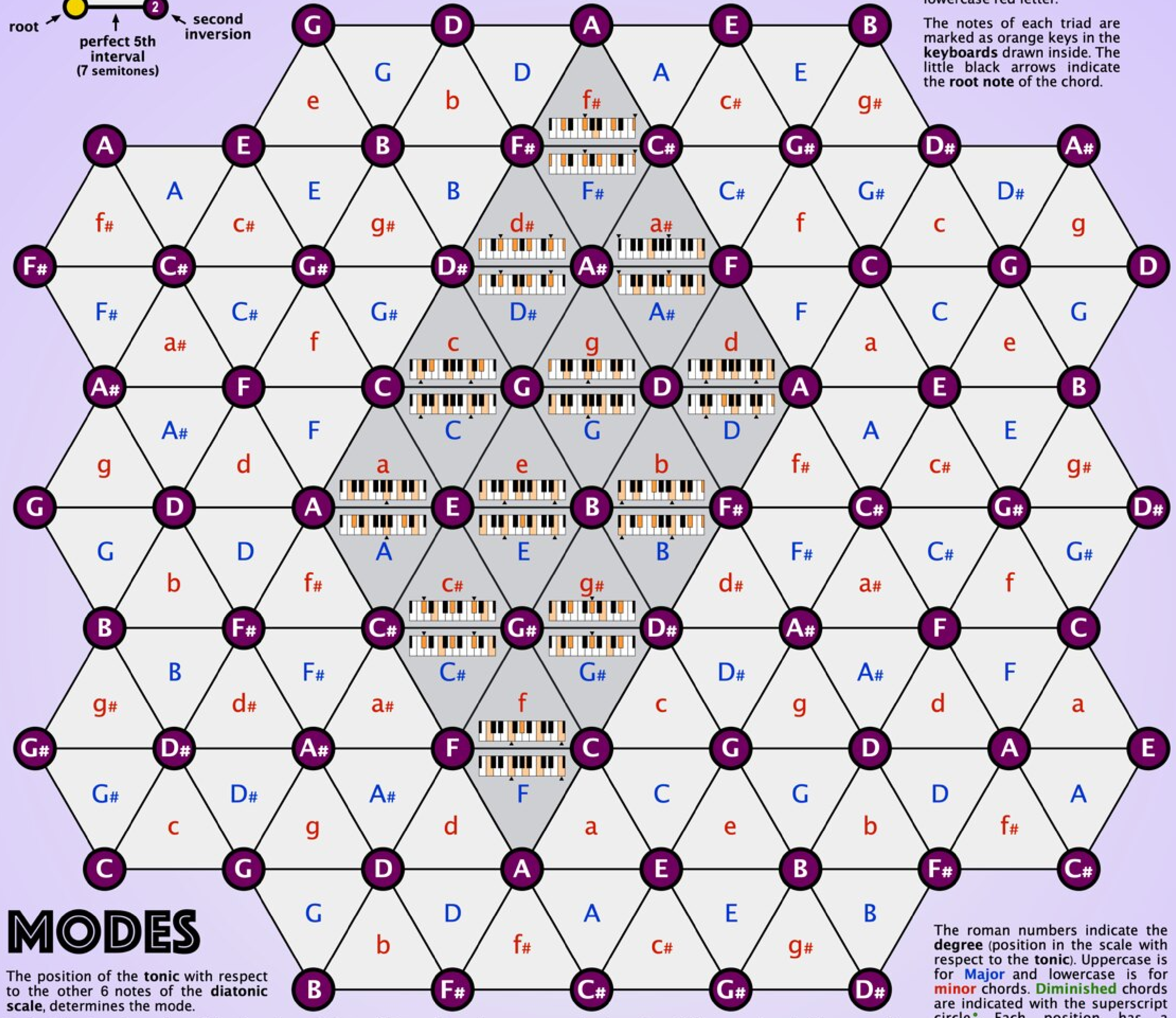




In 1739 the mathematician Leonhard Euler discovered that the 12 **notes** of the western **music system** can be arranged in an hexagonal grid, the **Tonnetz**. The notes are at the **nodes** of the grid (purple disks) and the interval between adjacent notes in each direction is the same across the entire diagram. For example, there is a perfect 5th interval between C and G, as well as between G and D. The shadowed parallelogram represents the **unit cell**, the block of notes that is repeated infinitely in both directions of the plane.

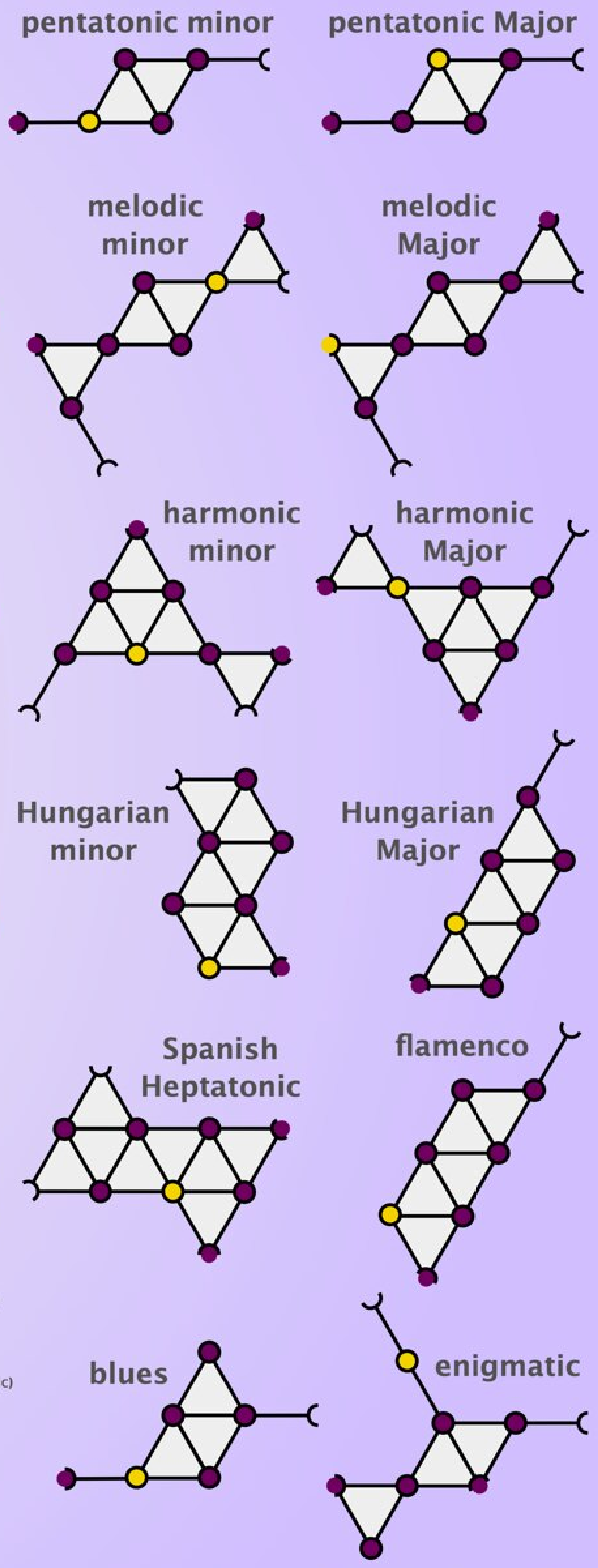
Each triangle in the Tonnetz is a **triad** (group of 3 notes), corresponding to either a **Major** chord, labelled with an uppercase blue letter, or a **minor** chord, labelled with a lowercase red letter.

The notes of each triad are marked as orange keys in the **keyboards** drawn inside. The little black arrows indicate the **root note** of the chord.



SCALES

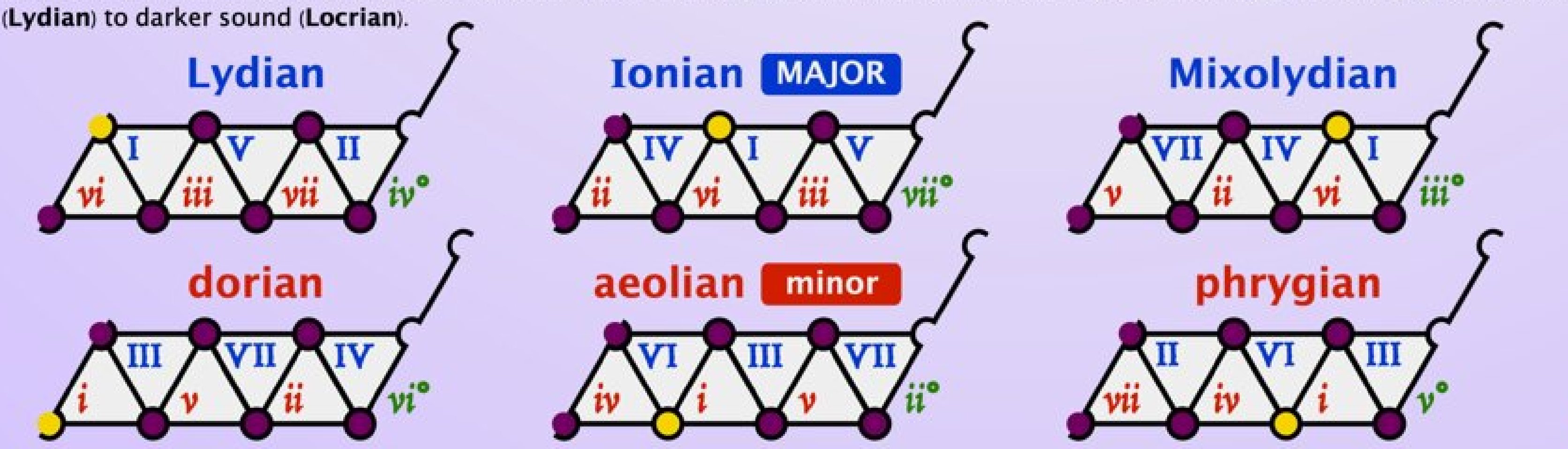
Any set of notes forms a **scale**. Scales are characterized by how many notes they have and by their relative positions. In the **Tonnetz**, each scale has a shape, and all the chords that fit into those shapes, are **chords** of the scale. The most common scale is the **diatonic** scale (see its 7 **modes** at the bottom). Here are some other common **scales**. Yellow disks mark the **tonic** note of the scale.



MODES

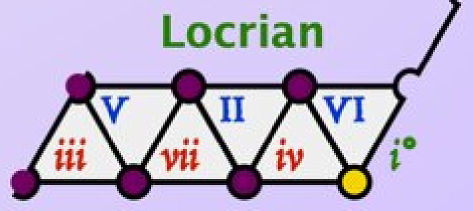
The position of the **tonic** with respect to the other 6 notes of the **diatonic scale**, determines the mode.

Each of the 7 main modes of the diatonic scale has a different flavor in western music since the middle ages, from brighter sound (**Lydian**) to darker sound (**Locrian**).

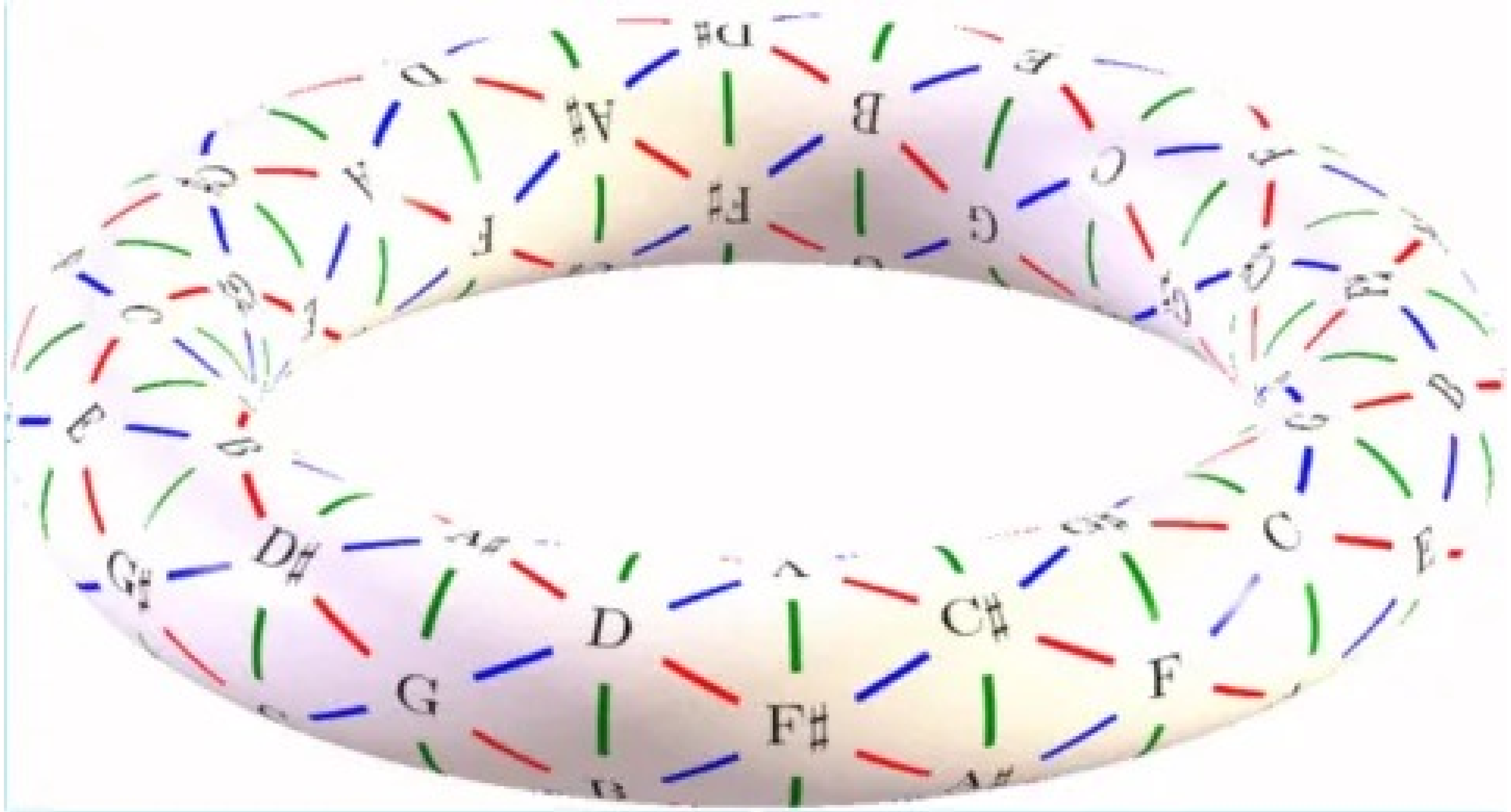


The roman numbers indicate the **degree** (position in the scale with respect to the **tonic**). Uppercase is for **Major** and lowercase is for **minor** chords. **Diminished** chords are indicated with the superscript circle°. Each position has a function in western tonal system:

- I *i* **Tonic** (centric note, where to resolve)
- II *ii* **Supertonic**
- III *iii* **Mediant**
- IV *iv* **Subdominant**
- V *v* **Dominant** (most important after tonic)
- VI *vi* **Submediant**
- VII *vii* **Subtonic** or **leading note**



© goodmoodgroup 2020



A♯
B
F♯
G
E
B
C
E
G♯
D♯
D
C♯
G
F
F♯